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Private interest in public tenders: no revenue, no efficiency and no social benefits

Final Report

Private Interests in Public Tenders

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Abstract

The paper explores auctions that assume additional conditions to be satisfied by the winner. If it is the task of the auctioneer to determine some specific characteristics of the good, his large discretion allows him to manipulate the results of the auction in his own interests. If the auctioneer is self-interested, he choses the 'most specific' conditions for the tender, while revenue maximization requires the 'least specific' conditions. The paper aims to provide a framework for understanding of allocative and revenue inefficieny of many auctions and tenders held in Russia during transition.

1. Introduction

When a good is to be auctioned off and it is the task of the auctioneer to determine some specific characteristics of the good, his large discretion allows him to manipulate the results of the auction. In particular, if the auctioneer is interested in getting side-transfers from bidders instead of revenue maximization, he may choose a particular pattern of characteristics to favor some bidder. These side-transfers (bribes) become potentially very large, when bidders' preferences differ substantially. In Russia and some other transition economies that have experienced a rapid privatization with very large and heterogeneous assets being privatized, this problem is particularly severe. When a large enterprise is privatized through an open auction, the requirement of large-scale investment favors liquidity-rich agents (outsiders), while the requirement to keep (excess) employment favors liquidity-constrained insiders. The same logic applies to many public tenders currently held in Russia and other economies in transition.

The Moscow Times, a respected Russian newspaper, notes that "the government's instructions on state contracts include clauses permitting both open and closed tenders. The latter could be used to place contracts in the "right," pre-determined hands, just the way 1995's loans-for-shares privatization scheme ended up being a parody of open bidding."[1] The very recent experience shows that the problem of designing tenders with special conditions is a real issue. In early 2000, the Ukrainian government announced an open tender to sell a control (55 percent of voting shares) of *Nikolaevsk Alumina Factory.* One of conditions specified was to built an additional alumina factory near-by. Interestingly, the *Siberian Aluminium Group,* the major aluminium producer in Russia (and one of the top ten in the world) and one of most-likely bidders for the *Nikolaevsk Alumina Factory,* had announced its plan to built such a factory *before* the Ukrainian government decided to held a tender.

Even in developed countries (e.g., France or Italy) corruption may be a real issue in public market auctions (Compte et al, 1999). In Russia, high discretion of bureaucrats, inconsistent and incomplete normative acts, inadequate tax legislation and possibility of collusion of agents in court procedures, make these issues much more serious. Also, normative acts issued at the federal and local levels often lack overall inner consistency. For the issue of corruption, it is worth to emphasize that inconsistency of normative acts creates discretion.

It should be noted that it is not necessarily the case that the auctioneer specifies conditions that lead to inefficiency in exchange for bribes. Allocation of the good to particular hands through a formally open may be due to some political reasons (see Boyko, Shleifer, and Vishny, 1993, and Shleifer, 1997), and, at least theoretically, be consistent with the social welfare maximization.

[1] *The Moscow Times*, April 18, 1997.

The theory being developed within this paper by no means applies to public auctions exclusively. It also gives non-trivial insights for almost any allocative mechanism. For instance, the war-of-attrition model, with its usual rent-seeking interpretation, might be modelled as an all-pay auction. Anticipating a formal model below, the main idea is as follows. The bureaucrat has a task to allocate a single indivisible good by an auction and, prior to the auction, to choose some condition α (alternatively, one may call α 'quality' of the good) from the set of possible conditions (alternatives) A to be satisfied by the winner. Given any α, agents assign some expected value for the good and compete in the auction.

The main model (Section 3 is based on Schwarz and Sonin, 2001; see also Schwarz, 2002) focuses on the possibility of achieving an efficient outcome in an environment that allows agents to receive both exogenous and endogenous shocks to their valuations and the auctioneer to alter rules of the game by providing favors to the bidder. It might seem a little bit restrictive, as compared to Section 2 approach, but there are some reasons to start with efficiency consideration. First, even the most opportunistic auctioneer, a pure bribe-taker, not necessarily opposes the goal of achieving efficiency. Often, it is an efficient outcome that generates maximum side-payments to the auctioneer, especially when it generates a maximum surplus to the winner. Also, although we do not prove it in this paper, there might be a result, analogous to the celebrated revenue-equivalence theorem, for the environment, and in the corresponding equilibrium the auctioneer receives all the 'credibility' payments

We consider a problem of designing an efficient mechanism for allocating the object in the environment, where signals about bidder's private values (s_i, received at the beginning if the game and t_i, received after actions are taken and objects are allocated) are private signals. A traditional Vickrey-Clarke-Groves mechanism cannot be straightforwardly employed since bidders do not know their exact valuations which are evolving over time and bidders can take value-enhancing actions. Given two-stage nature of information, efficiency requires two announcements of types. In contrast with some models with multi-dimensional types, the announcements cannot be made at the same time, due to dependence of the efficient investment decision on the realization of the preliminary signals.

First, we define an outcome as the identity of the bidder who receives the object and the list of private investments taken by bidders. For a given outcome, the social surplus is equal to the value of the object to the agent who receives it net of the total cost of investment. We consider a social planner as an efficiency benchmark. The social planner pursues a strategy that maximizes the expected social surplus. After the social planner observes the first-period signals s_i obtained by bidders, she has to decide which bidders have to receive favors (equivalently, what are the characteristics of the object to be sold) should act in the second period and which should abstain from actions. Since the exogenous shock of Stage 5 is not known at Stages 3 and 4 (when decisions to take actions are made), it may be efficient to have more than one bidder receiving a favor or taking an action or to have no bidders at all taking actions.[2] Part (i) of Theorem 4.1 establishes that if the social planner orders an agent with a value s_i to act, then she also orders all agents with value greater than s_i to act. Of course, an all-knowing and well-intentioned social planner is rarely available in the real world. What happens if there is no social planner but all the information is common knowledge, i.e. signals obtained by a bidder about her private value are observed by all players? Theorem 4.1 establishes that the efficient allocation can be achieved in a decentralized case.

The above mentioned results rely on bidders' private values being common knowledge. A more realistic case, where bidders privately observe their valuations, is of primary interest. Can an efficient allocation be achieved in that case? It is straightforward that an efficient allocation can not be attained without revelation of bidders' private signals prior to the moment, when decisions are to be made. If the object is allocated to the bidder with the highest value following revelation of final information (using, say, a second-price sealed-bid auction) adding a cheap talk stage following the initial period will not result in any information revelation and thus would lead to an inefficient outcome (allocation). In the cheap talk stage each bidder would claim to be 'the high type' because the higher is the perceived type of a bidder, the less likely are the other bidders to undertake actions and thus the lower are the subsequent bids for the object by other players. Theorem 4.4 and Theorem 4.7 show that there exists a mechanism, where private information is revealed and the object is assigned efficiently. First, after private signals s_i are received by agents, bidders reveal their private signals s_i by making payments (we show that the higher is the private signal s_i, the higher is the agent's willingness to pay for reporting to other agents that the value of her private signal s_i is high). Second, it consists of a second-price sealed-bid auction conducted after signals t_i are received. As long as private signals s_i are truthfully revealed in the first round, the subgame corresponding to the second round is identical to the complete information game. Theorem 4.4 establishes that the mechanism described above has an efficient separating equilibrium. Unfortunately, this mechanism also has an inefficient pooling equilibrium. To rule out the pooling equilibrium, we propose a class of mechanisms that force players to coordinate on the separating equilibrium. We refer to mechanisms from this class as "$\varepsilon-$efficient mechanisms." We prove that one can always choose an $\varepsilon-$efficient mechanism which yields an efficient allocation with probability arbitrarily close to one. An $\varepsilon-$efficient mechanism consists of two rounds. The first round takes place after private signals s_i are received by

[2] Indeed, for a given distribution of the Stage 5 exogenous shock, it becomes inefficient for anybody to undertake an action as the cost of action approaches the benefit. On the other extreme, if the cost of action approaches zero it becomes efficient for more than one agent to undertake an action.

agents: a non-transferable discount for amount ε is sold via a sealed-bid all-pay auction. After the all-pay auction all bids are made public. The ε discount can only be used in the second round auction. In the second round the object is sold using a Vickrey auction (if the winner of the Vickrey auction is a holder of the ε discount, she pays the second highest bid minus ε). For $\varepsilon = 0$, this mechanism is identical to the efficient mechanism described above. Theorem 5.3 shows that an arbitrarily small positive ε forces agents to coordinate on a separating equilibrium that yields an efficient allocation with probability converging to one as ε converges to zero. In spirit, this mechanism is very close to virtual implementation, a pure theoretical concept in mechanism design (e.g., Maskin and Sjostrom, 1999). The proposed mechanism is sufficiently straightforward and intuitive to have a chance of being useful in practice. It is possibly a first example of a virtual-implementation mechanism simple enough to be conceivably used in the real world.

A recent account of basic auction theory and an extensive literature survey could be found in Klemperer (1999). For the case of private values, the possibility to explore the Vickrey auction (instead of English or first-price sealed-bid auctions) is justified by the use of the revenue equivalence theorem (Myerson, 1981). Efficiency of generalized Vickrey auctions is proved in Dasgupta and Maskin (1998). Common value auctions are introduced in Milgrom and Weber (1982). Auctions for privatization are modelled in Maskin (1992, 2000). Effects of entry, which are closely related to the problem (as it can be seen from Example 2), were studied in Levin and Smith (1994), McAfee and McMillan (1987) and Riordan and Sappington (1987).

A major theoretical challenge is that a study of effects of bureaucrat's discretion calls for modelling agents with heterogenous tastes. However, many basic auction-theory results hold for homogenous agents only. In particular, the first-price sealed bid auction is no longer efficient[3] when bidders are heterogenous (Dasgupta and Maskin, 1998), and the revenue equivalence theorem no longer holds in its usual form (Myerson, 1981, Klemperer, 1999). Hopefully, the model would help in designing multidimensional (e.g., price-quality) auctions (cf. Che, 1993, and Branco, 1997).

The rest of the report is organized as follows. Section 2 introduces a simple theoretical model and derives policy implications. In Section 3, some extensions are discussed. Section 4 presents evidence, and Section 5 concludes. Appendix contains all proofs.

[3] An auction is *efficient* if it allocates the good to the buyer with the highest valuation. The second-price auction is always efficient in the case of private values.

2. The 2 Bidders Model

In this section, we develop a simple model with two bidders.[4] The object to be sold has a quality $\alpha \in A$. The auctioneer wants to allocate a good by an auction to maximize a social (or, in an alternative interpretation, the owner's) utility function $U_S = U_S(p, \alpha)$, where p is the price paid by the winner (i.e. the revenue) and α is the quality of the good sold.

There are two risk-neutral agents (participants). Each bidder i privately values a good with the quality α at $v_i(\alpha)$ drawn from a uniform distribution on the set $[0, V_i(\alpha)]$, functions $V_i(\alpha)$ being common knowledge. The formal procedure is as follows: the auctioneer (who has his own utility function U_B, possibly different from U_S) chooses and announces some α, then agents bid for the good, and the highest bidder pays the second price.

Assume now that the function $V_1(\alpha)$ decreases with α, while the function $V_2(\alpha)$ is an increasing function[5]. These assumptions reflect diversity of agents' preferences: the agent's 1 preferred choice (with respect to quality only) is $\alpha_1^* = 0$, while the agent's 2 preferred choice is $\alpha_2^* = 1$[6]. For the sake of simplicity, we maintain additional assumptions: the functions $V_i(\alpha)$ are continuous[7], and $V_1(0) > V_2(0)$ and $V_1(1) < V_2(1)$. These assumptions yield that there is a unique α such that $V_1(\alpha) = V_2(\alpha)$. However, the analysis can be carried on without changing results if they are relaxed.

Timing **Stage 1.** The bureaucrat chooses some $\alpha_b \in A$.

Stage 2. Agents independently learn their values $v_i(\alpha) \in [0, V_i(\alpha)]$ for each $\alpha \in A$ and determine bribes they are ready to pay for each α.

Stage 3. The bureaucrat announces $\alpha_b \in A$.

Stage 4. Agents bid for the good with characteristics α_b. The winning price is determined, and the winner gets the good and pays the price. If the bureaucrat is corrupt and the agent he favors wins, the bureaucrat receives a bribe. Otherwise, his pay-off is zero.

The auction at Stage 4 is organized as a Vickrey second-price sealed-bid auction: First, all participants independently submit their bids. Second, the highest bidder obtains the prize and pays the second highest bid. Let $R(\alpha)$ and $S_i(\alpha)$ denote the expected revenue of the auctioneer, i.e. the expected second price, and the expected surplus of the ith bidder, when α is chosen as the good's

[4] This game is analyzed in detail in Kutsevich (2003).

[5] If it is assumed, in addition, that the functions $V_i(\alpha)$ are differentiable, they satisfy the following properties: $V_1'(\alpha) < 0$, and $V_2'(\alpha) > 0$.

[6] Note that the analysis below is not consistent with the case when the agents' interests are aligned. More precisely, the general theory works if V_i are co-monotonic too, but all propositions should be stated quite differently.

[7] Clearly, continuity of value functions is a redundant assumption.

quality.

An important advantage of Vickrey auctions with private values is that for each bidder, it is a weakly dominant strategy to reveal truthfully her valuation (see Appendix A). This greatly simplifies all calculations. At the same time, the revenue equivalence theorem (see Appendix A) states that the expected revenue and pay-offs of participants are the same in a wide range of auctions with private values, including English and first-price sealed bid auctions. Thus, although actual auctions are usually English (ascending) or first-price sealed-bid, it is very instructive to study second-price Vickrey auctions.

The only purpose of the first result is to provide a benchmark for study of the case of a corrupt auctioneer.

Assume that the bureaucrat's and social interests are completely aligned, $U_B = U_S = p$, i.e. the auctioneer is benevolent and maximizes revenues; and the two bidders are symmetric, $V_2(1-\alpha) = V_1(\alpha)$. Then the optimal choice of the auctioneer is $\alpha^* = \frac{1}{2}$. This result essentially says that to maximize revenue, the auctioneer chooses the 'least specific' good.

Next, consider a more general example with heterogenous bidders.

Theorem 2.1. [8]. *Assume that $U_B = U_S = p$, and $V_1(\alpha)$ is a decreasing function, while $V_2(\alpha)$ is an increasing function. Then the optimal (revenue-maximizing) choice of the auctioneer is a unique α^* such that $V_1(\alpha^*) = V_2(\alpha^*)$.*

The expected payment of the winner (second-highest valuation when $\alpha = \alpha^*$) is equal to $\frac{1}{3}V_1(\alpha^*)$. In a more general case, $U_B = U_S = U(\alpha, p)$, the optimal choice of the auctioneer is, at the same time, the social choice. However, the higher is the difference between U_S and $U = p$ (the more strongly α matters), the less revenue is expected.

If the auctioneer is corrupt, the result may be completely different. Suppose that the auctioneer accept bribes in exchange for announcing some α. By assumption, the auctioneer has all the bargaining power over the bribe; that is, if some α' is announced in exchange for bribe $b = b(\alpha', j)$ from the agent j, then the bribe is equal to the expected payment of the agent j, when α' is announced[9].

Theorem 2.2. *Assume that the auctioneer is fully corrupt, i.e. is interested in the size of a bribe only, $U_B = b$. Then the auctioneer announces $\alpha_B^* \in \{0, 1\}$, with $\alpha_B^* = 0$ if $S_1(0) \geq S_2(1)$ and $\alpha_B^* = 1$, otherwise.*

[8] All proofs are given in Appendix B.

[9] If the bureaucrat and the winner divide the surplus by another rule (e.g., employ Nash bargaining solution), main results remain the same.

The expected bribe to be paid by the winner amounts to

$$\max\{S_1(0), S_2(1)\} = \max\left\{\frac{V_1(0) - V_2(0)}{2} + \frac{V_2^2(0)}{6V_1(0)}, \frac{V_2(1) - V_1(1)}{2} + \frac{V_1^2(1)}{6V_2(1)}\right\}.$$

The result is the opposite to the one of Theorem 2.1: to maximize side-payments, the auctioneer chooses a "most-specific" good. The intuition behind this result is straightforward: the higher is the difference between the bidders' expected valuation, the higher is the winner's expected surplus, that is, the amount she wants to pay as a bribe. In our simple setup, the boundary points ($\alpha \in \{0, 1\}$) are the ones with the highest difference between the bidders' interests. Thus, the auctioneer focuses exclusively on the two boundary points. In economic terms, the bureaucrat, once he decided on whose offer he accepts, tries to maximize the favorite's surplus by suppressing competition. Here, the possibility to push down competition is provided by the bureaucrat's discretion over the choice of α.

In the presence of corruption, expected revenues are sub-optimal. In the equilibrium of Theorem 2.2, the expected revenue of the owner is equal to $R(0) = \frac{V_2(0)}{2} - \frac{V_2^2(0)}{6V_1(0)}$ if $\alpha_B^* = 0$, and to $R(1) = \frac{V_1(1)}{2} - \frac{V_1^2(1)}{6V_2(1)}$ if $\alpha_B^* = 1$. Consider, for example, the case of $\alpha_B^* = 0$. If $V_1(0)$ is much higher than $V_2(0)$, which is not an unreasonable assumption in the point of maximum difference in interests, then the amount of side-transfer, $b = S_1(1) = \frac{V_1(1) - V_2(1)}{2} + \frac{V_2^2(1)}{6V_1(1)}$ is much higher than $R(0) = \frac{V_2(0)}{2} - \frac{V_2^2(0)}{6V_1(0)}$. If $V_2(0) = 0$, i.e. nobody *except* the winner, needs the good, the expected revenue is zero![10] This allows to understand the magnitude of losses in the case of corrupt tenders.

In real life, the auctioneer verifiable revenue would not be zero if a reserved price r is specified. In 1995 loans-for-shares auctions, the final price paid typically exceeded the reservation price by a small amount (e.g., \$170.1 mln instead of the reserved \$170 mln for *Norilsky Nikel* with the two loosing bids of \$170 mln). This comes at no surprise in the perfect-information world, but bidding close to the reservation price is highly unlikely to be an optimal strategy in a competitive environment with uncertainty.

Theorem 2.2 demonstrates that if the bureaucrat has large discretion over determination of the good's quality, and cares about side-payments only, the outcome might be tremendously inefficient. Also, onc show that if the public is able to reduce the bureaucrat's discretion, this reduces efficiency losses. Namely, let $D' \subset D$ be subsets of $A = [0, 1]$. Suppose that the auctioneer maximizes the difference between bidders valuations and the intersection of bidders valuation lies within D'. Then the expected revenue (social utility) will be higher and the expected bribe lower if the bureaucrat's choice is restricted to D', rather than to D. This explains the bureaucrat's willingness to have large discretion, since it brings him more side-payments. Thus, if the domain D itself is a matter

[10] Note that this result holds for all standard auctions, not only for the second-price sealed bid one.

of government choice, the bureaucrat will try to make it as large as possible. In reality, such an enlargement (an opportunity to impose more and more conditions) is often justified by necessity to maintain social stability/benefits if it is a liquidation procedure, or to defend national (regional, local) interests in a public tender. The message here is that this is usually done at the cost of efficiency.

In many cases, a more realistic assumption about behavior of the organizer of public tender is that he is neither benevolent ($U_B = U_S$), nor fully corrupt ($U_B = b$), but lies somewhere in-between. That is, $U_B = \lambda U_S + (1-\lambda)b$, where λ might reflect the extent of control (by public or by owner) over the auctioneer, $0 \leq \lambda \leq 1$. Let $|v|$ denote the absolute value of v.

Theorem 2.3. *Given any U_B and U_S, a higher level of control, λ, makes the social and bureaucrat's interests more aligned,* $\frac{d|U_B(\alpha^*)-U_S(\alpha^*)|}{d\lambda} < 0$.

2.1. Allocative Efficiency

Few words should be said about allocative efficiency when the auctioneer is corrupt.[11] In the case of two bidders considered above, the final allocation is always efficient: the bidder with the highest valuation wins the auction at Stage 4. However, this would not be the case if we consider an extension of the basic model with n heterogenous bidders. For $n = 3$ and the quality parameter, α, being still one-dimensional, one might consider the following example: Suppose, as above, that the function $V_1(\alpha)$ decreases with α, while the function $V_2(\alpha)$ is an increasing function. In addition, let $V_3(\alpha) = V_2(\alpha)$ for all α, and assume that $V_1(1) = V_2(0) = V_3(0) = 0$, and $V_1(0) < V_2(1)$. It is straightforward that a benevolent auctioneer would choose $\alpha^* = 1$, with the expected revenue equal to $\frac{V_2(1)}{3}$, while a corrupt auctioneer will choose $\alpha^*_B = 0$. Another possibility to get an inefficient allocation is to consider co-monotonic valuations (e.g., $V_1'(\alpha) > 0$, $V_2'(\alpha) > 0$ for all α).

2.2. Policy Implications

Before proceeding to a general model, which focuses on specific issues of efficient mechanism design in the environment, where the auctioneer can change parameters of the competition, we formulate some non-technical implications that follow from the simple model:

1. A benevolent auctioneer will specify conditions to enhance price competition of bidders. When the agent have different tastes satisfying the conditions above, the revenue-maximizing choice would be the 'least specific'.

[11] Recall that efficiency assumes that the bidder with the highest valuation obtains the good.

2. An auctioneer that cares about side-payments chooses the "most-specific" condition.

3. A tender held by a corrupt auctioneer might result in allocative inefficiency, as well as in losses in revenues.

4. The larger is the discretion of bureaucrats in determining the actual tender structure, the larger are efficiency losses.

5. Designing tenders to maintain social utility might result not only in inefficiency, but also in reduced social benefits.

3. The Main Model

3.1. The Environment

There are N identical agents. First, agents receive some private information about their private values of the object to be sold.[12] Then the auctioneer makes a decision that affects the value of the object to the bidders (provides favors, determines additional conditions, or specifies quality of the object to be sold). After that, each of the agents has an opportunity to take a costly action that increase her private value of the object. Then agents receive some additional information about the private value of the object to them.[13] Finally, an auction takes place.

Timing

Stage 1. Each agent receives a signal $s_i \geq 0$ about her private values (her capacity), drawn independently from the same atomless distribution $F(\cdot)$.

Stage 2. The lobbying (or, alternatively, a bribing) game takes place. During a game, some information (possibly, zero) is revealed. Below this game is described in detail.

Stage 3. The auctioneer chooses the individual-specific level of requirements $\boldsymbol{\theta} = (\theta_1, ..., \theta_N) \in \mathbf{R}^N$, $\sum \theta_i \leq D$.[14]

Stage 4. Each agent i has an opportunity to take an unobservable action, i.e. choose $a_i \in \{0, 1\}$, which increases the agent's private value by ba_i and costs $ca_i \geq 0$.[15]

[12] In particular, these first signals make agents ex-ante asymmetric.

[13] This structure does not necessarily assume some time passing. For example, the first signal might be interpreted as a raw estimate of the value, and the second one is a refined one.

[14] Alternative specifications are possible, e.g. one can assume that each vector $\boldsymbol{\theta}$ is associated with some cost $k(\boldsymbol{\theta})$ (see also Schwarz, 2002).

[15] When $a_i = 1$ we say that the agent i undertakes the action or simply 'acts'; if $a_i = 0$ we say the agent i abstains

Stage 5. Agents receive independent signals $t_i \geq 0$ about their private values. We assume that a higher Stage 1 signal s_i makes a higher second-period t_i more likely. Formally, if $s_i > s_i'$, then the distribution of t_i conditional on s_i stochastically dominates the distribution of t_i conditional on s_i'.[16]

Stage 6. The object is sold via second-price sealed-bid auction.[17]

Agent's i private value of the object equals $V_i = V(s_i, t_i)$, which depends on her first and Stage 5 signals plus the benefit from taking an action. Thus the utility of the agent is given by:

$$U_i = \begin{cases} V(s_i, t_i) + \theta_i + (b - c)a_i - p_i, & \text{if the agent } i \text{ wins the object} \\ -ca_i - p_i, & \text{otherwise,} \end{cases}$$

where p_i denotes the total amount of payments made by the agent i within a mechanism (i.e. not including c).[18] Note that p_i need not be equal to zero for loosing bidders. We assume that $V(s_i, t_i)$ is continuous and increasing in both arguments.

Social surplus is the value of the object to the agent that gets the object minus the cost of actions taken by all agents: $S = V(s_j, t_j) + \theta_i + ba_j - \sum_{i=1}^{N} ca_i$, where j is the identity of the agent that receives the object. An *allocation* is a vector consisting of the list of agents who took actions and the identity of the agent who received the object. An allocation needs to specify the identities of agents who took actions because actions affect the social surplus. An equilibrium strategy profile of a mechanism (e.g., an auction) is referred to as an *allocation rule.* If a mechanism has multiple equilibria, each equilibrium strategy profile defines an allocation rule.

Any allocation rule induces a probability distribution over values of social surplus induced by a mechanism or by a social choice rule adopted by the social planner. Allocation rules can be ranked in terms of efficiency by comparing corresponding expected values of the social surplus. An allocation rule is *efficient* (first-best), if it yields the same expected social surplus as the maximum expected social surplus that can be achieved by the social planner, who observes all signals received by agents, orders agents to take or not to take actions, and, finally, assigns the object.

The equilibrium concept we are mostly concerned with is a Bayesian perfect equilibrium. We will call it simply *equilibrium* in cases, where it leads to no ambiguity. Also, in most cases, where beliefs that support an equilibrium are self-evident, we do not specify them explicitly. Since we describe

from acting or skips the action. Obviously, only the case of $b > c$ is of interest.

[16]For instance, this condition holds if random variables s_i and t_i are affiliated (Milgrom and Weber, 1982), which includes independent variables as a particular case.

[17]Second-price sealed-bid auctions are a rare event in the real life. However, having such an auction at Stage 6 is strategically equivalent to having a usual open ascending-bid (English) auction.

[18]It is possible to extend our model to the case when the utility function takes the form $U_i(s_i, t_i, a_i)$, where a_i is continuous, and higher values of a_i makes a higher third-period signal t_i more likely. However, it would make exposition much more complex, while providing no new insight.

the lobbying game below, we do not give a unifying formal description of an equilibrium here. In the most important case, each equilibrium consists of a profile of reports $\widehat{s}_i$ (being functions of Stage 1 signals), payments h_i that make these reports credible and the corresponding beliefs, a favors profile and an action profile as functions of reports (and consistent with the beliefs), and dominant strategies at the auction of Stage 6.

For convenience of the reader, below we list notation used in the paper.

Notation

$\mathbf{s} = (s_1, ..., s_N) = (s_i, \mathbf{s}_{-i})$	agents' signals received at Stage 1
$\mathbf{t} = (t_1, ..., t_N)$	agents' signals received at Stage 5
$V(s_i, t_i)$	agent i's value to object
$\mathbf{a} = (a_1, ..., a_N)$	action profile, $a_i \in \{0, 1\}$
b	benefit from action (applies if the object is won)
c	cost of action
S	social surplus
G_i	change in expected social surplus due to i's action
g_i	change in agent's i expected pay-off due to action
$\mathbf{a}(m)$	actions profile: agents with m highest s_i's act
$\widehat{\mathbf{s}} = (\widehat{s}_1, ..., \widehat{s}_N) = (\widehat{s}_i, \widehat{\mathbf{s}}_{-i})$	agents' reports of their first-period signals
h_i, H_i	payments making agent's i report credible
$\pi_i(s_i, \hat{s}_i, \mathbf{s}_{-i})$	agent i's pay-off net of signaling costs
X, Y, Z	generic random variables (in Appendix)
$\boldsymbol{\theta} = (\theta_1, ..., \theta_N)$	vector of favors provided by auctioneer
p_i	agent i's payment within a mechanism

4. Efficient Mechanisms

In this section, we start with considering a benchmark case of the efficient mechanism for allocating the object that can be achieved by a social planner who knows all the private information available to bidders. In this case, the bureaucrat is essentially an all-knowing and benevolent social planner. Then we consider a mechanism that allocates the object efficiently in the incomplete information case.

4.1. The Social Planner's Problem

Let us start by characterizing the solution to the social planner problem. After observing the first signals, the social planner decides which agents should receive favors and what agents should act. Formally, there is a mapping of a vector of the first signals (received at Stage 1) into two vectors, a vector of favors $\boldsymbol{\theta}^*(\mathbf{s}) = (\theta_1, ..., \theta_N)$ and a vector of actions $\mathbf{a}^* = \mathbf{a}^*(\mathbf{s})$ at Stage 4.[19] At Stage 6, the social planner assigns the object, thus mapping a triplet of vectors $(\mathbf{s}, \mathbf{a}, \mathbf{t})$ into a number between 1 and N. The final assignment of the object is easily characterized: The social surplus maximization calls for assigning the object to the agent with the highest ex-post private value: if the efficient allocation assigns the object to the agent j, then for any $i \neq j$, we have $V(s_j, t_j) + \theta_j + ba_j \geq V(s_i, t_i) + \theta_i + ba_i$. Thus, assigning the object before agents have learned their final values of the object is likely to be inefficient. Obviously, in our environment, giving the object to the agent with the highest ex-post value is necessary, but not sufficient for efficiency. It remains to characterize the function $(\boldsymbol{\theta}^*(\mathbf{s}), \mathbf{a}^*(\mathbf{s}))$ that describes the favors and actions needed to maximize the expected social surplus, given $\mathbf{s}$. So, the social planner's problem might be written as follows:

$$\max_{\mathbf{a},\boldsymbol{\theta}} E_{\mathbf{t}}[S|\mathbf{s}, \mathbf{a}] = \max_{\mathbf{a},\boldsymbol{\theta}} \left\{ E_{\mathbf{t}} \max_i \{V(s_i, t_i) + \theta_i + ba_i\} - c \sum_{j=1}^{N} a_j \right\}.$$

Before proceeding to general results, let us illustrate this problem with a simple example.

Technically, it is useful to introduce a function $G_i(\mathbf{s}, \mathbf{a}_{-i})$ representing the difference in the expected social surplus that results from the agent i acting and not acting (keeping the actions of other agents unchanged):

$$G_i(\mathbf{s}, \mathbf{a}_{-i}) = E_{\mathbf{t}}[S|\mathbf{s}, \mathbf{a}_{-i}, a_i = 1] - E_{\mathbf{t}}[S|\mathbf{s}, \mathbf{a}_{-i}, a_i = 0]. \quad (4.1)$$

Here we assume that favors profile is given. (Later, we use this machinery to describe the optimal profile chosen by the planner.) Also, since the social planner maximizes social surplus, the expected surplus in the above formula should be computed under assumption that at the last stage the social planner allocates the object to the agent with the highest value. The social planner faces the following trade-off: each additional agent's act increases the expected private value of the agent who receives the object, but is associated with the cost of c. Let $\mathbf{a}(m) = \mathbf{a}(m, \mathbf{s})$ denote the vector of actions, where the agents with the highest m Stage 1 signals act, while the other $N - m$ agents skip action.

[19] In the most general case, the social planner may assign mixed strategies to the agents. We show later that almost surely, the social planner problem has a unique pure strategy solution. Consequently, we focus on pure strategies of the social planner.

Theorem 4.1. *For a given vector of Stage 1 private signals* $\mathbf{s}$ *and any profile of favors* $\boldsymbol{\theta}$*, there exists a threshold* $r^* = r^*(\mathbf{s})$ *such that the social planner assigns agents with the highest* r^* *Stage 1 signals to act.*[20,21]

Proof. To prove Theorem 4.1, we need to establish the following Lemma (a proof is relegated to the Appendix).

Lemma 1. *Consider vectors of actions* a *and* a' *such that* $\sum_i a_i = \sum_i a'_i$, $a_i = a'_i$ *for all* $i \neq j, k$*, and let* $a_j = 1$, $a_k = 0$, $a'_j = 0$, *and* $a'_k = 1$. *If* $s_j \geq s_k$*, then the expected social surplus from* a *is greater than that from* a'.

This Lemma shows that a vector of actions maximizing the expected social surplus must be of the form $\mathbf{a}(m)$ for some m, $0 \leq m \leq N$. Since there is a finite number of possible m's, there exists some r^* such that $\mathbf{a}(r^*)$ is the global maximizer of the expected social surplus. This completes the proof of Theorem 4.1.■

Next, we turn to a world without an all-knowing and well-intentioned social planner. Namely, we consider the case, where agents act non-cooperatively, given that Stage 1 signals are common knowledge. This is an essential step towards mechanism design for the incomplete information case.

One might expect that in the decentralized case too many or too few players may take actions, since they may not fully internalize the effect of their private actions on other players. We show that an efficient allocation can be achieved in a decentralized case, when bidders know each other's Stage 1 signals. Theorem 4.2 states that in this case there exists an equilibrium outcome of a second price auction conducted at Stage 6 period that yields an efficient allocation, the same allocation as the first best obtained by the social planner.

Theorem 4.2. *For any vector of favors* $\boldsymbol{\theta} = (\theta_1, ..., \theta_N)$*, if Stage 1 signals* $\mathbf{s}$ *are public knowledge, there exists a socially efficient (given* $\boldsymbol{\theta}$*) perfect Bayesian equilibrium of the game. In this equilibrium, agents take unobservable actions as if they were assigned by the social planner resulting in the allocation rule characterized in Theorem 4.1.*[22]

[20] r^* is determined almost uniquely: The event that the expected social surplus is maximized by more than one action vector of the form $\mathbf{a}(r^*)$ and $\mathbf{a}(r^{**})$ where $r^* \neq r^{**}$ has zero probability.

[21] Definition of $G_i(\mathbf{s}, \mathbf{a}_{-i})$ implies that an action vector maximizing the social surplus must satisfy $G_i(\mathbf{s}, \mathbf{a}_{-i}) \geq 0$ when $a_i = 1$ and $G_i(\mathbf{s}, \mathbf{a}_{-i}) \leq 0$ when $a_i = 0$.

[22] The equilibrium described in Theorem 4.2 seems to be a natural focal point. However, the game has a coordination component: there are other perfect Bayesian equilibria that are not efficient. For example, if there are only two players, there might be two equilibria: one with the highest-ranked agent acting and the other abstaining, and another one with the second-ranked agent acting and the highest-ranked abstaining.

The basic intuition is as follows: the expected increase in an agent's utility from taking an action is exactly equal to the change in the expected social surplus due to her action.[23] Then the fact that $\mathbf{a}(r^*)$ is the social planner's optimal choice ensures that $\mathbf{a}(r^*)$ is an equilibrium vector of actions in the non-cooperative game.

Proof of Theorem 4.2. First, we note that to prove all the assertions concerning favors profile $\boldsymbol{\theta}$, it is enough to redefine signals s_i and then leave the whole proof unchanged. Lemma A4 from the Appendix B shows that it is optimal for the social planner to allocate all favors to one bidder (a first-runner). (See Schwarz, 2002 for another proof.)

We introduce a function $g_i(\mathbf{s}, \mathbf{a}_{-i})$ defined as the change in the expected utility of the agent i as a result of taking an action instead of skipping it, and prove the following assertion (a proof is in the Appendix).

Lemma 2. $g_i(\mathbf{s}, \mathbf{a}_{-i}) = G_i(\mathbf{s}, \mathbf{a}_{-i})$.

Now observe that if $\mathbf{a}$ is a solution to the social planner's problem, then $G_i(\mathbf{s}, \mathbf{a}_{-i}) \geq 0$ when $a_i = 1$ and $G_i(\mathbf{s}, \mathbf{a}_{-i}) \leq 0$ when $a_i = 0$. Indeed, if $G_i(\mathbf{s}, \mathbf{a}_{-i}) < 0$ when $a_i = 1$, the agent's i switch from acting to non-acting would strictly increase the expected social surplus, contradicting the choice of $\mathbf{a}$. Similarly, $G_i(\mathbf{s}, \mathbf{a}_{-i}) \leq 0$ when $a_i = 0$. Then Lemma 2 asserts that for the change in private benefits we have $g_i(\mathbf{s}, \mathbf{a}_{-i}) \geq 0$ for agents that act, and $g_i(\mathbf{s}, \mathbf{a}_{-i}) \leq 0$ for others. Thus, no agent has incentives to deviate, and Theorem 4.2 is proven.■

Theorem 4.3. *(i) Suppose that the auctioneer believes that after the favors are allocated, the agents play the complete-information game as described in Theorem 4.2. Then, observing all Stage 1 signals, she allocates all the favors to one bidder with the highest Stage 1 signal.*

This theorem, though a straightforward corollary of Theorem 4.1, is an important observation leading to our main result. In the next section, we show that when Stage 1 signals are private information of agents, there exists a signalling mechanism that forces all agents to report their Stage 1 signals truthfully. Then, the auctioneer can makes her allocation of favors conditional on agents Stage 1 signals.

4.2. Efficient Lobbying

Now we are ready to investigate the incomplete information case. Here we consider a model similar to the one considered in the preceding section. The only (but crucial) difference here is that bidders'

[23] The logic behind the result is similar to the one that insures efficient entry in McAfee and McMillan (1987) and Levin and Smith (1994).

signals regarding their private values (**s** and **t**) are observed privately. Now a simple mechanism consisting of an auction conducted at Stage 6 no longer leads to an efficient allocation, since under such a mechanism the auctioneer allocates favors and agents take actions without knowledge of the private signals obtained by other players.[24] On the other hand, an efficient allocation rule can not always assign the final ownership of the object prior the end of Stage 5.

Is it possible to allocate an object efficiently in this environment, when both the auctioneer and agents have to face two problems (i) revelation of agents' private information and coordination and (ii) exogenous shocks represented by Stage 5 signal? This question is answered affirmatively by Theorem 4.4. We explicitly construct an efficient allocation mechanism. In doing this, we always assume that the auctioneer maximizes the expected social surplus, while allocating favors. Theorem 4.3 allows us to skip all the notation related to the allocation of favors in the subsequent analysis.

The Signalling Mechanism:

After Stage 1 (after the private signals **s** have been received by agents), all agents make simultaneous public announcements $\widehat{s}_i$ about their private values s_i. Then each agent voluntarily selects a payment amount, $h_i \geq 0$, that depends on the announcements of other agents, as well as on her own announcement. These payments $h_i(\widehat{\mathbf{s}})$ are necessary to make announcements credible, and actually are money-burning. Later we give the auctioneer a possibility to devise a special mechanism to collect the 'credibility payments'.

Theorem 4.4. *There exists a perfect Bayesian equilibrium of the Signaling Mechanism that yields a socially efficient allocation rule.*

In brief, Theorem 4.4 describes the following sequence of events. After agents learn their private signals, they make credible announcements about their private information. Then the auctioneer allocates all the favors to the agent with the highest signal. After that, some agents act to increase the value of the object for them conditional on winning, and some not. Then agents learn their refined signals and a second-price sealed-bid auction is conducted.

[24]For the sake of completeness, one can consider the no-signalling case, where an auction is conducted after the third period and no signaling takes place before the second period. (Note that cheap talk communication following the first stage is not credible because everybody has an incentive to exaggerate his signal.) To describe the symmetric equilibria of this game, one can show that there exists a unique constant s^* such that any agent acts if her first-stage value s_i is higher or equal to s^*, and abstains from acting otherwise. In the equilibrium, the expected number of actions is $N\left(1 - F_s(s^*)\right)$. So, in some cases, there are too few actions, while in others there are too many. This is a generalization of the Example from the Introduction. Also, there are a number of asymmetric equilibria. Of course, an asymmetric equilibrium can not lead to an efficient allocation rule.

Proofs of Theorem 4.4 and all subsequent results are relegated to the Appendix. Here, let us discuss the logic behind the result. First, note that if Stage 1 signals are revealed truthfully, the remaining subgame is identical to the game where Stage 1 signals $\mathbf{s}$ are common knowledge. Theorem 4.2 established that an efficient allocation is an equilibrium of that game. Consequently, in order to establish existence of an efficient allocation mechanism, it suffices to show that for some payment schedule $h_i(\widehat{\mathbf{s}})$, truthful reporting is an equilibrium, when agents anticipate that the equilibrium characterized in Theorem 4.2 will be played in the remaining subgame. The intuition behind the possibility of truthful revelation is as follows. The higher is Stage 1 signal s_i received by an agent i, the higher is that agent's relative willingness to pay in order to signal that her value of s_i is high. Agents are willing to pay in order to reveal their Stage 1 signals, because this information discourages other agents from taking actions, thus increasing the probability of winning for the agent i and decreasing the expected price that she will pay for the object (in the subsequent second price auction) conditional on winning. The expected price decrease affects agents with different private values differently. For instance, someone with a very low Stage 1 signal is unlikely to win the object, thus her willingness to pay for sending a signal that depress the price of the object is lower than that of an agent with a relatively high Stage 1 signal about her private value. This observation, which is critical to the existence of a separating signaling equilibrium, is formalized in Lemma *4.5*. This Lemma establishes an appropriate analog of the increasing-differences property (Milgrom and Shannon, 1994) for the pay-offs in the subgame.

Lemma 4.5. *Let $E\pi_i(s_i, \hat{s}_i, s_{-i})$ be bidder i's expected pay-off gross of $h_i(\widehat{\mathbf{s}})$, when her true private signal is s_i, while other agents believe that the vector of Stage 1 private signals is $(\hat{s}_i, s_{-i})$. For any s_{-i} and any $\hat{s}_i' > \hat{s}_i$, and any $s_i' > s_i$,*

$$E\pi_i(s_i', \hat{s}_i', \mathbf{s}_{-i}) - E\pi_i(s_i', \hat{s}_i, \mathbf{s}_{-i}) \geq E\pi_i(s_i, \hat{s}_i', \mathbf{s}_{-i}) - E\pi_i(s_i, \hat{s}_i, \mathbf{s}_{-i}). \tag{4.2}$$

In the above Lemma, $E\pi_i(s_i, \hat{s}_i, \mathbf{s}_{-i})$ is the expected pay-off of agent i in the mechanism described in Section **??**, when Stage 1 private signals are given by $(s_i, \mathbf{s}_{-i})$ and player i plays the best response to the action profile of players $-i$ given by $\mathbf{a}(r^*(\hat{s}_i, \mathbf{s}_{-i}))$. (The action profile $\mathbf{a}(r^*(\hat{s}_i, \mathbf{s}_{-i}))$ is characterized in Theorem 4.1.) Essentially, $E\pi_i(s_i, \hat{s}_i, \mathbf{s}_{-i})$ is the pay-off received by agent i in the subgame computed under an assumption that all first round announcements are believed to be truthful, and that agent i reported $\hat{s}_i$, while her true private value is s_i.

Lemma *4.5* states that the same change in announcement (from $\hat{s}_i$ to $\hat{s}_i'$) brings more in expected surplus to the agent with relatively high true signal, s_i'. Note that $E\pi_i(s_i, \hat{s}_i, \mathbf{s}_{-i})$ is not the same as the expected utility of agent i, because it does not include the payments h_i made in the first round of the mechanism. The agent's utility is given by $E\pi_i(s_i, \hat{s}_i, \mathbf{s}_{-i}) - h_i$. Thus, truthful reporting s_i

is consistent with an equilibrium, if there exists a payment schedule $h(\hat{s}_i, \mathbf{s}_{-i})$ such that incentive compatibility and individual rationality constraints are satisfied. Namely, for any agent i and all $(s_i, \hat{s}_i, \mathbf{s}_{-i})$ the payments should satisfy the following conditions:

$$
\begin{aligned}
E\pi_i(s_i, s_i, \mathbf{s}_{-i}) - h(s_i, \mathbf{s}_{-i}) &\geqslant E\pi_i(s_i, \hat{s}_i, \mathbf{s}_{-i}) - h(\hat{s}_i, \mathbf{s}_{-i}) \qquad \text{(IC)} \\
E\pi_i(s_i, s_i, \mathbf{s}_{-i}) - h(s_i, \mathbf{s}_{-i}) &\geqslant E\pi_i(s_i, \hat{s}_i = 0, \mathbf{s}_{-i}) \qquad \text{(IR)}
\end{aligned}
$$

Note that finding $h(\hat{s}_i, \mathbf{s}_{-i})$ that satisfies the above constraints is sufficient for proving the claim of Theorem 4.4. Such payment schedule $h_i(\hat{s}_i, \mathbf{s}_{-i})$ is characterized in Theorem 4.6. Before proceeding to Theorem 4.6, we need to introduce one more definition.

Consider the efficient allocation rule characterized in Theorem 4.1. It implies that for any vector of Stage 1 private signals $\mathbf{s}_{-i}$, there exists a sequence $0 = \bar{s}_i(k_i^*) \leq \bar{s}_i(k_i^* - 1) \leq ... \leq \bar{s}_i(1) \leq \bar{s}_i(0) < \infty$, where $\bar{s}_i(k)$ is defined to be the minimal type of i such that exactly k highest-ranked agents (different from the agent i herself) act in the subgame equilibrium described in Theorem 4.2. Let $k_i^* = k_i^*(0, \mathbf{s}_{-i})$ be the number of agents acting, when i has the lowest possible type (zero). Within each segment described above, an agent's i report is irrelevant to the other agents' decisions on whether or not to act.

As above, let $\mathbf{a}(m)$ denote the vector of actions, where the agents with the highest m Stage 1 signals act, while the other $N - m$ agents skip action. Note that $\mathbf{a}(m)$ is a function of the vector of Stage 1 signals $\mathbf{s}$.

Theorem 4.6. *The following payments $h_i(\hat{s}_i, \hat{\mathbf{s}}_{-i})$ are consistent with an efficient equilibrium of the Efficient Mechanism. For any i,*

$$
\begin{aligned}
h_i(\hat{s}_i, \hat{\mathbf{s}}_{-i}) &= 0, \text{ whenever } \bar{s}_i(k_i^*) \leq \hat{s}_i \leq \bar{s}_i(k_i^* - 1), \qquad (4.3) \\
h_i(\hat{s}_i, \hat{\mathbf{s}}_{-i}) &= h_i(\bar{s}_i(k), \hat{\mathbf{s}}_{-i}) + E\pi_i(\bar{s}_i(k), \mathbf{a}(k)) - E\pi_i(\bar{s}_i(k), \mathbf{a}(k+1)),
\end{aligned}
$$

whenever $\bar{s}_i(k) < \hat{s}_i \leq \bar{s}_i(k-1)$, $k < k_i^*$.

Theorem 4.6 shows that for any agent i, the payment schedule satisfies incentive compatibility and individual rationality (IC and IR, respectively) constraints. Then, if agents in the set $-i$ report their type truthfully, $\hat{\mathbf{s}}_{-i} = \mathbf{s}_{-i}$, the payment scheme for the agent i given by (4.3) induces her to report her type truthfully, $\hat{s}_i = s_i$. The proof of Theorem 4.4 is based on combining Theorem 4.2 and Theorem 4.6.

Proof of Theorem 4.4. Lemma *4.5* proves that the above payment schedule induces truthful reporting by agent i, provided that all other agents' reports are truthful. The beliefs supporting the equilibrium in the signaling stage are straightforward: if a payment by an agent i is defined by (4.3), then the agents Stage 1 signal is perceived to lie within the respective range. In the subgame that starts after Stage 1 signals are revealed, agents play according to the strategies described in Theorem 4.2.■

In the above equilibrium, each agent reports her type truthfully regardless of the other agents' types given that these types are reported truthfully.[25] This is a kind of an ex-post equilibrium (Perry and Reny,1999), where no agent regrets her announcement after learning the other agents' types; thus, this mechanism is similar in spirit to the well-known Vickrey-Clarke-Groves mechanism (e.g., Vickrey, 1961, Krishna and Perry, 1998). However, unlike the Vickrey-Clarke-Groves mechanism, this is a two-round mechanism, where payments made in the signaling round of the mechanism have no direct impact on allocating of the object — these payments influence the allocation of object indirectly by shaping beliefs about Stage 1 signals.

The mechanism described above provides an ex-post efficient ex-post equilibrium. In such an equilibrium, agents' payments may depend on the other agents' announcements. Below we show that the Signalling Mechanism described in the previous section also has an ex-ante efficient separating equilibrium. In this equilibrium, agents make no announcements (or make uninformative announcements) in the cheap talk stage of the mechanism. They simultaneously make publicly observable payments H_i; an agent decides on the payment size without knowing the private signals of other agents. We show that there exists a fully separating equilibrium where there is a unique payment corresponding to each private signal s_i. Consequently, agents no longer need to make announcements, because the announcements of their private signals are revealed in the size of payments they make.

Theorem 4.7. *There exists an ex-ante efficient perfect Bayesian equilibrium in the Signalling Mechanism, where agents simultaneously make payments $H(s_i)$ that depend only on their private information s_i. Equilibrium payments are given by $H_i(\hat{s}_i) = E_{\mathbf{s}_{-i}} h_i(\hat{s}_i, \mathbf{s}_{-i})$, where $h_i(\hat{s}_i, \mathbf{s}_{-i})$ are equilibrium payments defined in Lemma 4.5. The auctioneer allocates all the favors to a bidder with the highest payment $H(\cdot)$.*

Let us discuss the intuition behind the proof of Theorem 4.7. According to Theorem 4.2, an efficient allocation can be obtained when Stage 1 signals s_i are common knowledge. It remains to show that the signaling mechanism proposed above is incentive compatible when an efficient equilibrium is chosen in the subgame following the signaling stage. More formally we need to show that for any

[25] As usual, the revelation principle (Myerson, 1979) allows us to assume that agents report their types directly, rather than conveying information via a special set of signals.

$\mathbf{s}_{-i}$,

$$s_i \in \arg\max_{\hat{s}_i} \left\{ E_{\mathbf{s}_{-i},\mathbf{t}} \pi_i(s_i, \hat{s}_i, \mathbf{s}_{-i}) - H_i(\hat{s}_i) \right\}$$

(note that here expectation is taken with respect to $\mathbf{s}_{-i}$ and $\mathbf{t}$). This result is a straightforward corollary to the existence of an ex-post equilibrium established in Lemma *4.5*. Existence of this 'ex-ante' separating equilibrium essentially follows from the fact that if the agent's i truth-telling is a best reply to *any* vector $\mathbf{s}_{-i}$ of other agents' signals, than it is a best reply on the average as well.

Proof of Theorem 4.7. It suffices to observe that

$$s_i \in \arg\max_{\hat{s}_i} \left\{ E_{\mathbf{t}} \pi_i(\hat{s}_i, \mathbf{s}_{-i}) - h_i(\hat{s}_i, \mathbf{s}_{-i}) \right\}$$

for any $\mathbf{s}_{-i}$ and any s_i, and take sum over all $\mathbf{s}_{-i}$.

Then note that all $h_i(\hat{s}_i, \mathbf{s}_{-i})$ and thus the function $H(s_i)$ increase in the bidder's i Stage 1 signal s_i. This allows to use $H(s_i)$ to report the true value of s_i. Beliefs are straightforward, and the rest of the proof follows that of Theorem 4.4.■

We interpret the result of Theorem 4.7 as saying that if agents are able to make costly lobbying efforts, than the ultimate outcome is socially efficient. Agents are able to report credibly their prior information and that allows the auctioneer to allocate all the favors to a front-runner.

4.3. Inefficient Lobbying

An efficient allocation mechanism requires that the ownership of the object is assigned at Stage 6, and reporting Stage 1 signals prior to allocation of favors and the action stage. For the sake of completeness, one can consider the no-signalling case, where an auction is conducted after the third period and no signaling takes place before the second period. (Note that cheap talk communication following the first stage is not credible because everybody has an incentive to exaggerate his signal.) To describe the symmetric equilibria of this game, one can show that there exists a unique constant s^* such that any agent acts if her first-stage value s_i is higher or equal to s^*, and abstains from acting otherwise. In the equilibrium, the expected number of actions is $N\left(1 - F_s(s^*)\right)$. So, in some cases, there are too few actions, while in others there are too many. This is a generalization of the Example from the Introduction. Also, there are a number of asymmetric equilibria. Of course, an asymmetric equilibrium can not lead to an efficient allocation rule.

Example 4.8. *Let the number of participants be* $N = 2$, *assume that* s_i *are privately observed signals independently drawn from the uniform distribution on* $[0, 1]$. *For simplicity, assume that there* *is no third period signal,* $t_i \equiv 0$.[26] *In a symmetric equilibrium with no revelation of the first-period signals, each agent acts if her probability of winning conditional on her own type is higher than* $\frac{c}{b}$. *That is, agent* i *acts if* $s_i \geq s^* = \frac{c}{b}$. *If* $s^* = \frac{3}{4}$, *then with probability* $\frac{1}{16}$ *both agents act (which is inefficient), and with probability* $\frac{9}{16}$ *no agent acts (which is inefficient as well). Therefore, on average there are too few actions (*$\frac{1}{2}$ *instead of* 1*). If* $s^* = \frac{1}{4}$, *the situation is reverse: with probability* $\frac{9}{16}$ *both agents act, and with probability* $\frac{1}{16}$ *no agent acts. On average, there are too many actions (*$\frac{3}{2}$ *instead of* 1*). This is hardly surprising: without signaling, there are too few actions, when actions are relatively costly (*$\frac{c}{b} = \frac{3}{4}$*), and there are too many actions, when actions are relatively cheap (*$\frac{c}{b} = \frac{1}{4}$*).*

Now suppose that $N \geq 2$. *If there is no third-period uncertainty (*$t_i \equiv 0$*), then the social planner chooses exactly one agent to act – the one with the highest first-period signal. On the other extreme, if there is no first-period signal (*$s_i \equiv 0$*), and the cost of action is sufficiently cheap, then the social planner would assign all agents to act.*

The same thing happens if lobbying efforts of an agent are hidden from other bidders. This illustrates why corruption (i.e. a situation where bribes are unobservable) is inefficient as compared to lobbying with observable efforts. The above Example can be readily extended as follows.

Theorem 4.9. *If lobbying efforts are unobservable (e.g. as a result of putting additional restrictions on the belief space), there is no efficient perfect Bayesian equilibrium.*

5. Auctioning Favors

The mechanism described in the previous section has two stages: first, each agents spends resources to persuade others that he has a high signal. Second, they decide whether or not to undertake actions (make investments) basing upon the information they inferred from the lobbying efforts of other agents. Here we show that the lobbying stage, where agents spend resources to reveal their types, might be replaced by a sealed-bid all-pay auction, where the object being sold is worth some additional favor $\varepsilon > 0$. In most of the subsequent analysis, it does not assumed that ε need to be small. The only result that requires the discount to be small is the one that shows that when ε approaches zero, the allocation mechanism becomes almost efficient. Because of this latter feature, we will generally refer to such a mechanism as an $\varepsilon-$*efficient mechanism* .

[26]In the special case of all t_i's equal to zero, an efficient allocation rule can be implemented by assigning the ownership of the object by conducting a Vickrey auction at the end of the first period after s_i's are privately learned. Also note that for any non-degenerate distribution of third-period signals, assigning the ownership of the object at the end of the first period is no longer efficient. Of course, the inefficiency of allocating the object at the end of the third period demonstrated by the example does not go away when t_i's are not equal to zero.

An $\varepsilon-$*efficient mechanism* has another advantage (Schwarz and Sonin, 2001). Theorem 4.7 established existence of an efficient perfect Bayesian equilibrium of this two stage auction. Unfortunately, this is not a unique equilibrium: a pooling equilibrium, where everybody bids zero in the signaling stage, is a natural focal point. Nevertheless, introducing an arbitrarily small inefficiency into the auction design can force bidders to coordinate on an efficient separating equilibrium. Finally, in the previous section, the costs of making reports credible have not been captured by the auctioneer. There is no reason why bidders would opt to announce their types by writing checks to the auctioneer, and not by burning money in some other way.

We start with describing an $\varepsilon-$efficient mechanism and then proceed to establish efficiency properties of this mechanism in Theorem 5.3.

Rounds of $\varepsilon-$Efficient Mechanism

1. The first (reporting) round takes place at the end of Stage 1 (after the private signals $\mathbf{s}$ have been received by agents, but before agents take actions). In this round one coupon is sold via all pay sealed bid auction.[27] All bids are announced at the end of the round. The coupon sold in the signaling round entitles its owner to a discount of size ε for the price in the final auction (the discount coupon is non-transferable, only the winner of the final auction can benefit from having the coupon).

2. The second round (final) auction takes place at the end of Stage 5, after agents observe private signals $\mathbf{t}$. In the second round the ownership of the object is assigned using a second-price sealed-bid auction, as prescribed by our procedure. (If the highest bidder in the final round is the owner of the $\varepsilon-$coupon, then she pays the second highest bid minus ε.)

There are two rounds and three decision nodes in an ε-efficient mechanism. At the first decision node, agents make bids in an all-pay auction, i.e. the i's actions space is $\{H_i|H_i \geq 0\}$. The information set of agent i at the first decision node is given by s_i. The first round strategy is described by the probability distribution $\rho_i(\cdot; s_i)$ over the set of pure strategies $\{H_i|H_i \geq 0\}$. At the second decision node, agents make a decision to act or not to act. The information set of agent i at the second decision node is given by $(s_i, H_i, \mathbf{H}_{-i}, \mathbf{w})$, where $\mathbf{w}$ is an N-dimensional vector with $w_k = 1$ if the agent k won the coupon in the all pay auction, and $w_k = 0$ otherwise. (There is a unique vector $\mathbf{w}$ consistent with vector of payments $\mathbf{H}$, unless there is a tie). The probability that agent i acts ($a_i = 1$) is denoted by $\lambda_i = \lambda_i(s_i, H_i, \mathbf{H}_{-i}, \mathbf{w})$. At the third decision node, agents submit bids in the second price sealed-bid auction. At this moment, the information sets are $(s_i, H_i, \mathbf{H}_{-i}, \mathbf{w}, a_i, t_i)$. It is

[27] In an all pay sealed bid auction every agent submits a sealed bid. All agents have to pay the amount of their bids regardless of whether or not they won the object. The agent with the highest bid receives the object. (In case of a tie the winner is randomly chosen from the set of highest bidders.) Fullerton and McAfee (1999) use an all-pay auction in their 'contestant selection auction'.

well known that in an equilibrium in weakly dominant strategies of a private value Vickrey auction bidders bid their true values. Thus, equilibrium bids are given by $V(s_i, t_i) + ba_i + \varepsilon w_i$.

Clearly, an ε-efficient mechanism has multiple equilibria. Some of these equilibria are highly implausible. In order to rule out such equilibria we introduce a restriction on strategies in the spirit of 'intuitive' criteria such as D1 of Cho and Kreps (1987) or stability of Kohlberg and Mertens (1986).

Definition 5.1. *A strategy of an agent j is monotonic, if two vectors $\mathbf{H}_{-j}$ and $\mathbf{H}'_{-j}$ differ only in component i so that $H_i > H'_i$, then $p_j(s_j, H_j, \mathbf{H}_{-j}, \mathbf{w}) \leq p_j(s_j, H_j, \mathbf{H}'_{-j}, \mathbf{w}')$.*

In words, a monotonic strategy of an agent j assumes that for any history, the probability that the agent j takes an action is non-increasing in the size of the payment that some agent i, $i \neq j$ makes in the signaling stage.

As we will see, the requirement that the strategies are monotonic rules out the 'bizarre' equilibrium, where all agents bid zero in the signaling stage and an agent who bids a positive amount is perceived to be of the lowest type. Basically, there are two reasons why an equilibrium strategy may not be monotonic: First, perverse beliefs may sustain an equilibrium in strategies that are not monotonic. An example of such 'unnatural' beliefs is as follows: The more an agent bids for a discount coupon, the lower is her perceived s_i. Obviously, this is counter-intuitive: the higher is an agent's s_i, the more she values the discount coupon. The second possibility stems from coordination aspect of the game. If bids in the signaling stage are used as coordination devices for selecting a perfect Bayesian equilibrium in the remaining subgame, an equilibrium resulting from these beliefs may include strategies that are not monotonic.

Definition 5.2. *A robust equilibrium of an $\varepsilon-$efficient mechanism is any symmetric perfect Bayesian equilibrium in monotonic strategies.*

Theorem 5.3. *For an ε-efficient mechanism, the following is true:*

(i) There exists a robust equilibrium.

(ii) The robust equilibrium is unique.

(iii) The probability that the robust equilibrium yields an efficient allocation converges to one as $\varepsilon \to 0$.

Before proceeding to a formal proof, let us sketch the intuition behind this result. A pooling equilibrium where everybody bids zero for the coupon is not robust. Indeed, if everybody bids zero for the discount, it can be purchased for an arbitrarily small amount. Thus, the pooling equilibrium is sustainable only if bidders are discouraged from bidding a positive amount by a belief that a positive bid would encourage other bidders to act more aggressively in the action stage. However, this belief is inconsistent with strategies being monotonic. The same argument applies to any partially pooling equilibrium. We show that there are no equilibria in mixed strategies, because the willingness to pay for the discount is an increasing function of the bidder's signal. Efficiency of a robust equilibrium follows from Theorem 4.7 that establishes that for $\varepsilon = 0$, there exists an efficient symmetric equilibrium. To prove asymptotic efficiency of a robust equilibrium, we show that when ε approaches 0, the robust equilibrium converges to the equilibrium described in Theorem 4.7.[28]

Proof of Theorem 5.3.

(i) The proof of existence follows the pattern of the proof of Theorem 4.4. Construction of an ex-ante equilibrium in the previous section used existence of an ex-post equilibrium in a mechanism, where credibility payments are allowed to be functions of announcements. Here, we use the same idea. As an intermediate step, consider a mechanism, where the discount is not auctioned off using an all-pay auction. Instead, bidders announce their types in the reporting stage (much like in the mechanism described in Section 4). After the announcement, bidders make payments $h_i(\widehat{s}_i, \widehat{\mathbf{s}}_{-i})$ to make the announcement credible, and the bidder with the highest announced s_i receives the $\varepsilon-$discount.

Rounds of the "intermediate" mechanism:

1. After each agent privately learns s_i, all agents simultaneously announce their types in the cheap talk stage. Afterwards, each agent must make a payment of $h_i(\widehat{s}_i, \widehat{\mathbf{s}}_{-i})$. The agent with the highest Stage 1 announcement $\widehat{s}$ receives the discount coupon (ties are broken using a lottery). Agents take action after observing announcements $\widehat{\mathbf{s}}$.

2. After Stage 5 signals $\mathbf{t}$ are revealed, the object is sold via a second-price sealed-bid auction.

We shall show that there exists a payment schedule $h_i(\widehat{s}_i, \widehat{\mathbf{s}}_{-i})$ such that truthful reporting supported by paying $h_i(\widehat{s}_i, \widehat{\mathbf{s}}_{-i})$ is an ex-post equilibrium. The private value of the bidder with the highest Stage 1 signal is essentially boosted by the amount equal to the discount ε. Let $\widetilde{\mathbf{s}}(\mathbf{s},\widehat{\mathbf{s}})$ be a vector of 'adjusted' private value signals, where $\widetilde{s}_i = s_i + \varepsilon$ if $\widehat{s}_i > \widehat{s}_j = s_j$ for all $j \neq i$, and $\widetilde{s}_i = s_i$ otherwise. Assuming $\widehat{\mathbf{s}}_{-i} = \mathbf{s}_{-i}$, we study the ith agent incentives to misreport the true signal s_i. If all the equilibrium reports $\widehat{s}_i$ are truthful, then the subgame after ε discount is assigned is identical to the game considered in Section 4.1. The equilibrium expected pay-off of agent i of the subgame, which does not include h_i, is denoted by $E\widetilde{\pi}_i(s_i, \hat{s}_i, \mathbf{s}_{-i})$. One can express $E\widetilde{\pi}_i(s_i, \hat{s}_i, \mathbf{s}_{-i})$ in terms of $E\pi_i(s_i, \hat{s}_i, \mathbf{s}_{-i})$ (defined in Lemma *4.5*) using 'adjusted' private signals. Let $\widehat{\widetilde{\mathbf{s}}}$ denote a vector of perceived 'adjusted' signals of agents; the ith component of $\widehat{\widetilde{\mathbf{s}}}$ is $\widehat{\widetilde{s}}_i = \widehat{\widetilde{s}}_i(s_i, \widehat{s}_i, \widehat{\mathbf{s}}_{-i}) = \widetilde{s}_i + (s_i - \widehat{s}_i)$. That is, $\widetilde{\mathbf{s}}$ is a vector of 'adjusted' private value signals and $\widehat{\widetilde{\mathbf{s}}}$ is public perception about $\widetilde{\mathbf{s}}$. Now we

[28] Also, if the ε-efficient mechanism yields an inefficient outcome, efficiency losses are of magnitude ε.

can write $E\widetilde{\pi}_i(s_i, \hat{s}_i, \mathbf{s}_{-i}) = E\pi_i(\widetilde{s}_i, \widehat{\widetilde{s}}_i, \widetilde{\mathbf{s}}_{-i})$.

To prove that a separating equilibrium exists, we need to formulate an increasing-differences condition similar to (4.2).

Claim. *For any $N-1$-tuple of truthful reports $\mathbf{s}_{-i}$, and any $\hat{s}_i' \geq \hat{s}_i$, $s_i' \geq s_i$,*

$$E\widetilde{\pi}_i(s_i', \hat{s}_i', \mathbf{s}_{-i}) - E\widetilde{\pi}_i(s_i', \hat{s}_i, \mathbf{s}_{-i}) \geq E\widetilde{\pi}_i(s_i, \hat{s}_i', \mathbf{s}_{-i}) - E\widetilde{\pi}_i(s_i, \hat{s}_i, \mathbf{s}_{-i}) \tag{5.1}$$

To prove the claim, we need to consider three cases: (a) the agent wins the ε discount if she makes announcement $\hat{s}_i'$ but not $\hat{s}_i$; (b) an agent wins the discount for either announcement $\hat{s}_i'$ or $\hat{s}_i$; (c) neither $\hat{s}_i'$, nor $\hat{s}_i$ are high enough to win the discount.

For (b) and (c), (5.1) follows immediately from Lemma *4.5*. It remains to show that it also holds for the case (a). Denote $\mathbf{s}_{-i} = (s_{-i}^m, \mathbf{s}_{-i}^{-m})$, where s_{-i}^m is the largest component of the vector $\mathbf{s}_{-i}$ and $\mathbf{s}_{-i}^{-m}$ is an $N-2$-dimensional vector that consists of all components of vector $\mathbf{s}_{-i}$ other than its largest component s_{-i}^m. Applying the new notation, one gets $E\widetilde{\pi}_i(s_i, \hat{s}_i, \mathbf{s}_{-i}) = E\widetilde{\pi}_i(s_i, \hat{s}_i, s_{-i}^m, \mathbf{s}_{-i}^{-m})$. In case (a), we have $\widetilde{\mathbf{s}}_{-i}^{-m} = \mathbf{s}_{-i}^{-m}$. Therefore, one can re-write (5.1) as follows:

$$E\pi_i(s_i' + \varepsilon, \hat{s}_i', s_{-i}^m) - E\pi_i(s_i', \hat{s}_i, s_{-i}^m + \varepsilon) \geq E\pi_i(s_i + \varepsilon, \hat{s}_i', s_{-i}^m) - E\pi_i(s_i, \hat{s}_i, s_{-i}^m + \varepsilon). \tag{5.2}$$

Let

$$\begin{aligned}
X &= V(s_i, t_i) + \varepsilon - \max_{j \neq i} \left\{ V(s_j, t_j) + b a_j^*(\hat{s}_i', s_{-i}^m, \mathbf{s}_{-i}^{-m}) \right\}, \\
X' &= V(s_i, t_i) + \varepsilon - \max_{j \neq i} \left\{ V(s_j, t_j) + b a_j^*(\hat{s}_i, s_{-i}^m + \varepsilon, \mathbf{s}_{-i}^{-m}) \right\}, \\
Y &= V(s_i', t_i) + \varepsilon - \max_{j \neq i} \left\{ V(s_j, t_j) + b a_j^*(\hat{s}_i', s_{-i}^m, \mathbf{s}_{-i}^{-m}) \right\}, \\
Y' &= V(s_i', t_i) + \varepsilon - \max_{j \neq i} \left\{ V(s_j, t_j) + b a_j^*(\hat{s}_i, s_{-i}^m + \varepsilon, \mathbf{s}_{-i}^{-m}) \right\}
\end{aligned}$$

We know that $X' \succeq X$, $Y' \succeq Y$. Then

$$\begin{aligned}
E\pi_i(s_i' + \varepsilon, \hat{s}_i', s_{-i}^m) - E\pi_i(s_i + \varepsilon, \hat{s}_i', s_{-i}^m) &= EY^+ - EX^+, \\
E\pi_i(s_i', \hat{s}_i, s_{-i}^m + \varepsilon) - E\pi_i(s_i, \hat{s}_i, s_{-i}^m + \varepsilon) &= EY'^+ - EX'^+.
\end{aligned}$$

Using Lemma A3 (from the Appendix) completes the proof of (5.2).

Since (5.2) holds, there exists an ex-post separating equilibrium in the "intermediate mechanism". Using existence of an ex-post equilibrium, we can apply the same argument as in the proof of Theorem 4.7 to establish existence of ex-ante separating signaling mechanism, where agents make signaling payments that are strictly increasing in their signals. This completes the proof of existence.

Now we shall prove that any robust equilibrium is unique, fully separating, and 'almost efficient'.

In an equilibrium, the probability of any particular bid value H in the signaling stage is zero. Indeed, if there is a positive mass of agents that plays some H_{mass} with positive probability, then there is a positive probability of a tie. Then an agent playing H_{mass} can increase the likelihood of winning the discount $\varepsilon > 0$ by increasing her bid by an infinitesimal amount. Since the strategies are monotonic, none of the agents would increase their likelihood of taking actions. Thus, such a deviation would be profitable.

Probability that players in the set $-i$ take actions is denoted here as $\mathbf{p}_{-i}$. Let $\Pi(s_i, \mathbf{p}_{-i}, \mathbf{s}_{-i})$ denote the pay-off of player i in the subgame after signaling payments H's are sunk. We want to show that if $\mathbf{p}_{-i} \geq \mathbf{p}'_{-i}$ then for every $s'_i > s_i$ we have

$$\Pi(s'_i, \mathbf{p}_{-i}, \mathbf{s}_{-i}) - \Pi(s_i, \mathbf{p}'_{-i}, \mathbf{s}_{-i}) \leq \Pi(s'_i, \mathbf{p}_{-i}, \mathbf{s}_{-i}) - \Pi(s_i, \mathbf{p}'_{-i}, \mathbf{s}_{-i}). \tag{5.3}$$

Essentially this condition says that any decrease in "final" private values of player in the set $-i$ is more valuable for player i with a larger Stage 1 private signal. Inequality (5.3) follows from the proof of Lemma *4.5*.

Let us show that all robust equilibria are separating. In a robust equilibrium, actions taken by players depend on their private signals and the announcements of other players. Thus, we can write $\mathbf{p}_{-i} = \mathbf{p}_{-i}(\mathbf{s}_{-i}, \mathbf{H}_{-i}, H_i)$ and $\mathbf{p}'_{-i} = \mathbf{p}_{-i}(\mathbf{s}_{-i}, \mathbf{H}_{-i}, H'_i)$. (According to Step 1 a tie is a measure zero event; and thus have no impact on expected payoffs.) For monotonic strategies $\mathbf{p}_{-i} \geq \mathbf{p}'_{-i}$ for $H'_i > H_i$ (the inequality holds for all components). Inequality (5.3) implies that $H(s)$ is weakly increasing in s. Combining this fact with result of Step 1, we conclude that $H(s)$ is strictly increasing in s (expect perhaps for a measure-zero set).

Let us show that in equilibrium, $p_i(\mathbf{H}_{-i}, H_i(s_i), s_i)$ is non decreasing in s_i. Indeed, $\mathbf{p}_{-i} = \mathbf{p}_{-i}(\mathbf{s}_{-i}, \mathbf{H}_{-i}, H_i)$ is weakly decreasing in H_i. Thus, according to single crossing condition, if agent with a Stage 1 signal s_i acts with positive probability $p_i(\mathbf{H}_{-i}, H_i(s_i), s_i) > 0$, then any agent with a signal $s'_i > s_i$ strictly prefers to act, and $p_i(\mathbf{H}_{-i}, H_i(s'_i), s'_i) = 1$. Therefore, there exists a unique equilibrium in the subgame that is consistent with a robust equilibrium strategy profile. In this equilibrium, all agents with private values exceeding some critical value $s^*(\mathbf{H})$ act.

From the previous paragraph and Theorem 4.2, it follows that $\varepsilon-$efficient mechanism yields an efficient allocation with probability converging to one as ε converges to zero.

To establish uniqueness of the robust equilibrium, we use a standard argument (e.g., Klemperer, 1999). Condition (5.3) implies that $\frac{dH(s)}{ds}$ is the same in any robust equilibrium. In Step 5, we showed that there is a unique robust equilibrium in the subgame following the all-pay auction. It remains to show that $H(0) = 0$. Suppose otherwise, say $H(0) = H_0 > 0$. For a player with $s_i = 0$,

$H(0) = 0$ is a profitable deviation: Indeed, after this she does not change the perception of her type (she is correctly perceived to have $s_i = 0$). It was demonstrated that in a robust equilibrium each player either acts with probability one or zero (except perhaps for a set of measure zero). Thus, the deviation can only cause other players to increase the probability with which they act; however, given the set of players that act, non of the players that do not act in a robust equilibrium would choose to act. ■

Let us now consider an example illustrating that the all pay auction part of the ε-efficient mechanism is crucial for ensuring that any robust equilibrium is separating and nearly efficient.

Example 5.4. *Suppose the all-pay auction is replaced with a second-price sealed-bid auction. When a sufficiently small discount is auctioned off via a second price auction, the following inefficient pooling equilibrium is robust: all agents bid ε for the discount of size ε. Indeed, we need to specify beliefs that support this equilibrium. If an agent deviates by bidding less than ε, she is perceived to have the lowest possible signal s_i. Thus, there are no incentives to bid less than ε, provided that ε is sufficiently small. If an agent bids more than ε, the beliefs of other agents about her type are the same as if she bids ε. Thus, bidding more than ε is a bad strategy: If there are N agents bidding ε each in a second-price auction, each of them has a $\frac{1}{N}$ chance of getting the discount. The winner of the discount "envy" the bidders who did not win the discount, and thus do not have to pay anything in the signaling stage. By bidding more than ε, an agent insures that she wins the discount and will have to pay for it, thus, making herself worse off. In contrast, there are no robust pooling equilibrium of the ε-efficient mechanism (by Theorem 5.3). For instance, if all agents bid ε for the discount, bidding slightly more than ε is a profitable deviation.*

6. Conclusion

The paper develops an auction-theory type model to demonstrate sources of inefficiency when the bureaucrat's interests are different from the social ones. In the model, the main source of inefficiency is the bureaucrats large discretion over parameters of the auction, and the possibility that his private interests are not fully aligned with public interests. If the auctioneer is not benevolent (does not maximize proceeds from the auction), he exploits his discretion to obtain 'non-competitiveness' rents. If the society has a possibility to reduce his discretion or introduce some external control over his action, the revenue (or, alternatively, the social utility) rise, while the rent (bribes) of the bureaucrat is reduced.

The consideration above is not complete, however. It seems that the model allows to obtain some additional insights. In particular, in the current paper, the usual rent-seeking interpretation of the all-pay auctions has not been employed. It would be extremely interesting to learn that the above mechanism may allow to achieve simultaneously: (i) spending-maximization in an all-pay auction (the bureaucracy side) and (ii) market-sharing (the large bidders side). The parties loosing are public (the state) and small bidders.

The main model in this paper is focused on the auctioneer trying to achieve efficiency. However, a revenue-plus-bribe-maximizing auctioneer might opt for another strategy.

Procurement auctions. At least five years ago, it was suggested to allocate all public procurement contracts in Russia by public tenders. The main concern was possibility of corruption when it is bureaucrat's own discretion to allocate the good. Although corruption need not necessarily challenge the allocative efficiency, it is harmful for cost minimization (revenue maximization in selling-the-good terms).[29]

Privatization and Liquidation Auctions. Many privatization auctions in Russia (both cash and non-cash) were organized in a way, where the winner had to fulfill some conditions (e.g., to make a specified investment or to keep some level of employment and/or output) to obtain the good. From the theoretical point of view, providing such an opportunity may result in revenue or social inefficiency. The possibility to determine some specific conditions while holding a liquidation auction (as specified by Russian bankruptcy law) may lead to huge efficiency losses, as it allows the liquidation manager to manipulate the results of the auction. This in turn affects both *ex-ante* and *ex-post* incentives of management and creditors. Cornelli and Felli (1999) try to design a bankruptcy procedure that maximize creditors (*ex-post*) returns. The model below might be used to demonstrate that it is done at a cost of efficiency. Also, the model sheds some light on mechanisms of 'targeted sales' in liquidation auctions. (See Lambert, Sonin, and Zhuravskaya, 2000, for a thorough study of bankruptcy in Russia.)

Collusion, Package-Bidding and Market-Sharing. In auction theory literature, collusion among bidders is considered as a major obstacle to revenue-maximization in auctions. (See a survey of collusion-in-auctions literature in Lyk-Jensen, 1998.) The proposed framework provides additional (and complementary) insights by noting that to have a bidding ring of colluding bidders, there should be a restriction on free entry to the auction. The possibility to specify conditions is the most natural way of restricting entry. (Compte et al, 2000 and Lambert and Sonin consider a particular case.) In a variety of cases, the main problem is that the package bidding is left to a bureaucrat. The approach of this paper allows to study effects of discretion in package bidding on allocative efficiency and revenue maximization. Indeed, if the set of all possible partitions (packages) constitute the set of conditions

[29] Note that in case of a procurement contract, all results should be re-stated in terms of costs instead of values, payments from the government instead of prices payed for the good being allocated, etc.

under which one single good is sold, then the problem of packaging is effectively transferred to the problem of choosing such a condition.

Lobbying and rent-seeking. Lobbying activities are often modelled as a game of attrition (e.g., Klemperer, 1999). More broadly, lobbying and rent-seeking may be considered as an all-pay auction (see definition below). Here the bureaucrat might be interested in total-spending maximization (i.e. not only in the winner's payment maximization). In a corruption-type interpretation, the bureaucrat chooses a set of rules (regulation) that maximize spending of bidders.

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Appendix A: The Simple Model

Proof of Theorem 2.1.

To simplify notation, let $a = V_1(\alpha)$ and $b = V_2(\alpha)$.

Assume that x and y drawn from distributions with c.d.f. F_x and F_y on $[0, a]$ and $[0, b]$, respectively, and let $a \leq b$. Then the first order statistics (i.e. the second price in the case of two bidders) has the following c.d.f.: $F_{\xi_{(1)}} = 1 - (1 - F_x)(1 - F_y)$. Thus, the expected revenue of the auctioneer is

$$E\xi_{(1)} = \int_0^a z dF_{\xi(1)}(z).$$

If $F_x(z) = \frac{z}{a}$ on $[0, a]$ and $F_y(z) = \frac{z}{b}$ on $[0, b]$ (i.e. both distributions are uniform, but domains are generally different), then

$$E\xi_{(1)} = \int_0^a \frac{z}{a} + \frac{z}{b} - \frac{2z^2}{ab} dz = \frac{a}{2} - \frac{a^2}{6b}.$$

Suppose that $a = a(\alpha)$ and $b = b(\alpha)$, $\alpha \in [0, 1]$ and $a'(\alpha) < 0$, while $b'(\alpha) > 0$. Denote by α^* a unique point of intersection, $a(\alpha^*) = b(\alpha^*)$. (Recall that we assumed that there is a unique point of intersection.)

The auctioneer solves the problem:

$$E\xi_{(1)} = E\xi_{(1)}[\alpha] \to \max_{\alpha}$$

The derivative of $E\xi_{(1)}$ w.r.t. α is

$$\frac{dE\xi_{(1)}}{d\alpha} = \frac{a'}{2} - \frac{2aa'b - b'a^2}{6b^2}.$$

Note that $\frac{dE\xi_{(1)}}{d\alpha} > 0$ for all $\alpha \leq \alpha^*$. Thus, α^* is a unique solution for the revenue maximization problem given $a \leq b$. The other case ($a > b$) is similar. Thus, α^* is a unique solution for all a, b.

When $\alpha = \alpha^*$, the expected revenue is

$$R(\alpha^*) = E\xi_{(1)} = \frac{a}{2} - \frac{a^2}{6b} = \frac{1}{3}a = \frac{1}{3}V_1(\alpha^*).\blacksquare$$

Proof of Theorem 2.2.

Expected (ex-ante) surplus of the bidder with the larger domain ($[0, b]$) is equal to

$$S_b = E(y - x \mid x < y)P(x < y) = \int_0^a \int_t^b z dF_y(z) dF_x(t) - \int_0^b \int_0^z t dF_x(t) dF_y(z).$$

It is straightforward that $S_b > S_a = E(x - y \mid x > y)P(x > y)$. Given that x and y are drawn from uniform distributions, one gets $E(y \mid x < y)P(x < y) = \frac{b}{2} - \frac{a^2}{6b}$ and $E(x \mid x < y)P(x < y) = \frac{a}{2} - \frac{a^2}{3b}$, summing up to

$$S_b = \frac{b-a}{2} + \frac{a^2}{6b}.$$

Clearly, $\frac{\partial S_b}{\partial b} > 0$ for all $b > a > 0$. If $\alpha < \alpha^*$, then $V_1(\alpha) > V_2(\alpha)$ and thus the above formula applies. Therefore,

$$S_1(\alpha) = \frac{V_1(\alpha) - V_2(\alpha)}{2} + \frac{V_2^2(\alpha)}{6V_1(\alpha)}$$

is maximized when $\alpha = 0$. Similarly, $S_2(\alpha)$ is maximized when $\alpha = 1$. Hence, the self-interested auctioneer is concerned with $\alpha \in \{0, 1\}$ exclusively.■

Theorem 2.3 is a straightforward calculation.

Appendix B: The Main Model

To prove propositions in the body text, we need some auxiliary notation and lemmas. For any number (function) x, let $x^+ = \max\{x, 0\}$. A random variable X (first-order) *stochastically dominates* a random variable Y (denoted $X \succeq Y$) if and only if for cumulative density functions, one has $F_X(z) \le F_Y(z)$ for any $z \in \mathbf{R}$. An equivalent condition is that $Eh(X) \ge Eh(Y)$ for any increasing function h (e.g., Levy, 1992).

Lemma A1. *Suppose that* X, Z and Y, W *are random variables, and in both pairs variables are independent of each other. Suppose that* $X \succeq Y$, $W \succeq Z$. *Then* $X - Z \succeq Y - W$.

Proof. We need to prove that for any t, $F_{X-Z}(t) \le F_{Y-W}(t)$. One has

$$\begin{aligned} F_{X-Z}(t) &= \int_{-\infty}^{\infty} \left[\int_{x-t}^{\infty} dF_Z(z) \right] dF_X(x) = \int_{-\infty}^{\infty} (1 - F_Z(x-t)) dF_X(x) \\ &\le \int_{-\infty}^{\infty} (1 - F_W(x-t)) dF_X(x) = \int_{-\infty}^{\infty} \left[\int_{x-t}^{\infty} dF_W(w) \right] dF_X(x) \\ &= \int_{-\infty}^{\infty} \left[\int_{-\infty}^{z+t} dF_X(x) \right] dF_W(w) = \int_{-\infty}^{\infty} F_X(w+t) dF_W(z) \\ &\le \int_{-\infty}^{\infty} F_Y(w+t) dF_W(z) = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{w+t} dF_Y(y) \right] dF_W(z) = F_{Y-W}(t). \end{aligned}$$

■

Lemma A2. *For any random variables* X *and* Y *such that* $X \succeq Y$, *and a random variable* Z, *which is independent of* X, Y,

$$\max\{X, Z\} \succeq \max\{Y, Z\}.$$

Proof. Straightforward.■

Lemma A3. *For any random variables* X *and* Y *such that* X *stochastically dominates* Y, *and any constant* $z \ge 0$,

$$E(X+z)^+ - EX^+ \ge E(Y+z)^+ - EY^+.$$

Proof. For any $z \ge 0$, the function $h_z(x) = (x+z)^+ - x^+$ is a bounded increasing function of x. Therefore, the definition of stochastic dominance yields that $Eh_z(X) \ge Eh_z(Y)$.■

Lemma A4. *For any independent random variables* X, Y, Z *such that* $X \succeq Y$, *and any constant* $t \ge 0$,

$$E \max\{X+t, Y, Z\} \succeq E \max\{X, Y+t, Z\}.$$

Proof. For any numbers x, y, and z, $\max\{x, y\} = (x-y)^+ + y$. We start with the following identites

$$\begin{aligned} \max\{X+t, Y, Z\} &= (X+t-\max\{Y, Z\})^+ + \max\{Y, Z\}, \\ \max\{X, Y, Z\} &= (X-\max\{Y, Z\})^+ + \max\{Y, Z\}. \end{aligned}$$

Then

$$\begin{aligned} \max\{X+t, Y, Z\} - \max\{X, Y, Z\} &= (X+t-\max\{Y, Z\})^+ - (X-\max\{Y, Z\})^+, \\ \max\{X, Y+t, Z\} - \max\{X, Y, Z\} &= (Y+t-\max\{X, Z\})^+ - (Y-\max\{X, Z\})^+. \end{aligned}$$

From Lemma A2, we know that $\max\{X, Z\} \succeq \max\{Y, Z\}$. Lemma A1 implies that $X - \max\{Y, Z\} \succeq Y - \max\{X, Z\}$. Using Lemma A3 completes the proof. ■

Lemma A5. *Let $q(x, y)$ be a continuous function increasing in both arguments, and let X, Y be two random variables. For any realizations $x_1 > x_2$ of the random variable X, the distribution of Y conditional on x_1 (first-order) stochastically dominates the distribution of Y conditional on x_2. Then $q(x_1, Y) \succeq q(x_2, Y)$.*

Proof. Define $\tau(x, z)$ to satisfy $q(x, \tau(x, z)) = z$. Clearly, $\tau(x, z)$ is increasing in z. Now $F_{q(x_1,Y)}(z) = F_{Y|x_1}(\tau(x_1, z)) \leq F_{Y|x_1}(\tau(x_2, z))$ and $F_{q(x_2,Y)}(z) = F_{Y|x_2}(\tau(x_2, z)) \geq F_{Y|x_1}(\tau(x_2, z))$, the latter inequality following from the fact that $Y|x_1 \succeq Y|x_2$. Therefore, for any z, $F_{q(x_1,Y)}(z) \leq F_{q(x_2,Y)}(z)$. ■

Proof of Lemma 1. Let $\widetilde{\mathbf{a}}$ be a vector of actions with $\widetilde{a}_j = \widetilde{a}_k = 0$ and $\widetilde{a}_i = a_i = a_i'$ for all $i \neq j, k$. Lemma A4 yields that $V(s_j, T_j) \succeq V(s_k, T_k)$ whenever $s_j \geq s_k$. Now one can use Lemma A3 with the constant $b\widetilde{a}_j = b\widetilde{a}_k$.■

Proof of Lemma 2. Let $Z = \max_{j \neq i} \{V(s_j, t_j) + ba_j\})$, and $X = V(s_i, t_i)$. By definition,

$$g_i(\mathbf{s}, \mathbf{a}_{-i}) = E(X+b-Z)^+ - E(X-Z)^+.$$

Using the formula $\max\{x, y\} = (x-y)^+ + y$, we get

$$\begin{aligned} G_i(\mathbf{s}, \mathbf{a}_{-i}) &= E\max\{X+b, Z\} - E\max\{X, Z\} \\ &= E(X+b-Z)^+ + EZ - (E(X-Z)^+ + EZ) \\ &= E(X+b-Z)^+ - E(X-Z)^+ = g_i(\mathbf{s}, \mathbf{a}_{-i}), \end{aligned}$$

as claimed.■

Proof of Lemma 3. First, we claim that for any i, and for any $\mathbf{s}$ and $\tilde{\mathbf{s}}$ such that $\mathbf{s}_{-i} \leq \tilde{\mathbf{s}}_{-i}$ and $s_i = \tilde{s}_i$, $a_i^*(\tilde{\mathbf{s}}) \leq a_i^*(\mathbf{s})$. Indeed, let $X_i(\mathbf{s}) = V(s_i, t_i)$ and $Z_i(\mathbf{s}) = \max_{j \neq i} \left\{V(s_j, t_j) + ba_j^*\right\}$.

Suppose that $a_j^*(s_i, \mathbf{s}_{-i}) \le a_j^*(s_i, \tilde{\mathbf{s}}_{-i})$ for all $j \neq i$. Then

$$X_i(s_i, \mathbf{s}_{-i}) - Z_i(s_i, \mathbf{s}_{-i}) \succeq X_i(s_i, \widetilde{\mathbf{s}}_{-i}) - Z_i(s_i, \widetilde{\mathbf{s}}_{-i})$$

by Lemmas A1 and A2. To prove that $g_i(\mathbf{s}) \ge g_i(\tilde{\mathbf{s}})$, recall that

$$g_i(\mathbf{s}) = E\left(X_i(\mathbf{s}) + b - Z_i(\mathbf{s})\right)^+ - E\left(X_i(\mathbf{s}) - Z_i(\mathbf{s})\right)^+,$$

and then apply Lemma A3 to prove the claim. By definition, $g_i(\mathbf{s}) \ge g_i(\tilde{\mathbf{s}})$ implies that $a_i^*(\tilde{\mathbf{s}}) \le a_i^*(\mathbf{s})$. It is enough to consider the case of $a_j^*(s_i, \mathbf{s}_{-i}) \le a_j^*(s_i, \tilde{\mathbf{s}}_{-i})$ for all $j \neq i$. Indeed, if a switch from 1 to 0 occurred with an agent that ends up higher than i as a result of increase from $\mathbf{s}_{-i}$ to $\tilde{\mathbf{s}}_{-i}$, then it is definite that $a_i^*(\tilde{\mathbf{s}}) = 0$, and thus $a_i^*(\tilde{\mathbf{s}}) \le a_i^*(\mathbf{s})$ for any $a_i^*(\mathbf{s})$. Otherwise (if a change have occurred with an agent ranked lower than the agent i), $a_i^*(\mathbf{s}) = 1$.

Second, we claim that the function g_i increases with s_i. The first claim shows, in particular, that if s_i increases, while $\mathbf{s}_{-i}$ is constant, the number of agents acting (weakly) decreases. Thus, the random variable $X_i(s_i, \mathbf{s}_{-i}) - Z_i(s_i, \mathbf{s}_{-i})$ raises in terms of stochastic dominance, and Lemma A3 applies.

Now we shall prove that

$$E\pi_i(s_i', \hat{s}_i') - E\pi_i(s_i, \hat{s}_i') \ge E\pi_i(s_i', \hat{s}_i) - E\pi_i(s_i, \hat{s}_i),$$

which is equivalent to (4.2).

Define $X_i = V(s_i, t_i)$, $X_i' = V(s_i', t_i)$, $Y_i = \max_{j \neq i}\left\{V(s_j, t_j) + b a_j^*(\hat{s}_i, \mathbf{s}_{-i})\right\}$, and $Y_i' = \max_{j \neq i}\left\{V(s_j, t_j) + ba\right.$

$$\begin{aligned} E\pi_i(s_i', \hat{s}_i) - E\pi_i(s_i, \hat{s}_i) &= E(X_i' - Y_i)^+ - E(X_i - Y_i)^+, \\ E\pi_i(s_i', \hat{s}_i') - E\pi_i(s_i, \hat{s}_i') &= E(X_i' - Y_i')^+ - E(X_i - Y_i')^+, \end{aligned}$$

and so it remains to prove that

$$E(X_i' - Y_i')^+ - E(X_i - Y_i')^+ \ge E(X_i' - Y_i)^+ - E(X_i - Y_i)^+.$$

The two claims proved above yield that $X_i' \succeq X_i$ and $Y_i \succeq Y_i'$. Using Lemma A3 (for each non-negative constant) completes the proof.■

Proof of Theorem 4.6. Let s_i be the true agent's i first-period signal, and consider k such that $\bar{s}_i(k) < \hat{s}_i \le \bar{s}_i(k-1)$. Since $\hat{\mathbf{s}}_{-i}$ is fixed throughout the argument, we suppress the notation. Truthful reporting brings the expected utility of

$$E\pi_i(s_i, \mathbf{a}(k)) - h_i(s_i) = E\pi_i(s_i, \mathbf{a}(k)) - h_i(\bar{s}_i(k)) - E\pi_i(\bar{s}_i(k), \mathbf{a}(k)) + E\pi_i(\bar{s}_i(k), \mathbf{a}(k+1)).$$

First, we prove that the agent i has no incentives to under-report her first-period signal, i.e. to report $\hat{s}_i < s_i$. Consider incentives the agent i with the first-period signal $\bar{s}_i(k)$ faces. For any ε such that $\bar{s}_i(k) - \bar{s}_i(k+1) > \varepsilon > 0$, she is indifferent between reporting $\bar{s}_i(k)$ and reporting $\bar{s}_i(k) - \varepsilon$. Indeed, the 'credibility payment' is the same and the number of acting rivals is the same $(k+1)$. The condition (4.2) assures that if the agent with $\bar{s}_i(k)$ is indifferent between reporting $\bar{s}_i(k)$ to reporting $\bar{s}_i(k) - \varepsilon$, then the agent with $s_i > \bar{s}_i(k)$ (weakly) prefers reporting $\bar{s}_i(k)$ to reporting $\bar{s}_i(k) - \varepsilon$. Thus, $\hat{s}_i$ can not be less than $\bar{s}_i(k)$. (To rule out reports below $\bar{s}_i(k+1)$, one can consider incentives the $\bar{s}_i(k+1)$-agent faces.) It remains to show that $\hat{s}_i$ (weakly) exceeds $\bar{s}_i(k)$. So, we need to prove that

$$E\pi_i(s_i, \mathbf{a}(k)) - h_i(\bar{s}_i(k)) - E\pi_i(\bar{s}_i(k), \mathbf{a}(k)) + E\pi_i(\bar{s}_i(k), \mathbf{a}(k+1)) \geq E\pi_i(s_i, \mathbf{a}(k+1)) - h_i(\bar{s}_i(k)),$$

or equivalently,

$$E\pi_i(s_i, \mathbf{a}(k)) - E\pi_i(s_i, \mathbf{a}(k+1)) \geq E\pi_i(\bar{s}_i(k), \mathbf{a}(k)) - E\pi_i(\bar{s}_i(k), \mathbf{a}(k+1)),$$

but this is true by (4.2). Since the agent i having the signal s_i is indifferent between reporting s_i and reporting any signal that is larger than $\bar{s}_i(k)$ and does not exceed s_i, the proof that the agent i has no incentives to under-report her signal is complete.

The proof that there is no incentives to over-report the signal is somewhat symmetric. The s_i-agent is indifferent between reporting the true signal and reporting $\bar{s}_i(k-1)$. Indeed, the mechanism assumes that the agents with reports s_i and $\bar{s}_i(k-1)$ pay the same amount. Now, for any ε such that $\bar{s}_i(k-2) - \bar{s}_i(k-1) > \varepsilon > 0$, the agent with $\bar{s}_i(k-1)$ is indifferent between reporting the true signal and reporting $\bar{s}_i(k-1) + \varepsilon$. To see this, note that

$$\begin{aligned} E\pi_i(\bar{s}_i(k-1), \mathbf{a}(k)) - h_i(\bar{s}_i(k-1)) &= E\pi_i(\bar{s}_i(k-1), \mathbf{a}(k-1)) - h_i(\bar{s}_i(k-1)) \\ &\quad - E\pi_i(\bar{s}_i(k-1), \mathbf{a}(k-1)) + E\pi_i(\bar{s}_i(k-1), \mathbf{a}(k)). \end{aligned}$$

By (4.2),

$$E\pi_i(\bar{s}_i(k-1), \mathbf{a}(k-1)) - E\pi_i(\bar{s}_i(k-1), \mathbf{a}(k)) \geq E\pi_i(s_i, \mathbf{a}(k-1)) - E\pi_i(s_i, \mathbf{a}(k)).$$

Thus, if the $\bar{s}_i(k-1)$ is indifferent between reporting the truth and reporting $\bar{s}_i(k-1) + \varepsilon$, the s_i-agent (weakly) prefers to report $\bar{s}_i(k-1)$ (which is pay-off equivalent to reporting the truth), than to report $\bar{s}_i(k-1) + \varepsilon$. To show, that $\hat{s}_i$ would not exceed $\bar{s}_i(k-2)$, one should consider the incentives the $\bar{s}_i(k-2)$-agent faces, etc. Therefore, the agent i has no incentives to over-report her first-period signal.■