Structural breaks and financial risk management
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Abstract

There is ample empirical evidence on the presence of structural changes in financial time series. Structural breaks are also shown to contribute to the leptokurtosis of financial returns and explain at least partly the observed persistence of volatility processes. This paper explores whether detecting and taking into account structural breaks in the volatility model can improve upon our Value at Risk forecast. VAR is used by banks as a standard risk measure and is accepted by regulation in setting capital, which makes it an issue for the central bank guarding against systemic risk.

This paper investigates daily BUX returns over the period 1995-2002. The Bai-Perron algorithm found several breaks in the mean and volatility of BUX return. The shift in the level of unconditional mean return around 1997-1998 is likely to be explained by the evolving efficiency of the market, but most of all by the halt of a strong upward trend in the preceding period. Volatility jumped to very high levels due to the Asian and Russian crisis. There were longer lasting shift too, most likely due to increasing trading volume. When in-sample forecasts are evaluated, models with SB dummies outperform the alternative methods. According to the rolling-window estimation and out-of-sample forecast the SB models seem to perform slightly better. However the results are sensitive to the evaluation criteria used, and the choice on the probability level.

JEL classification numbers: G10, G21, C22, C53

Keywords: Structural Break tests, volatility forecasting, Value-at-Risk, backtest
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Introduction

The aim of this paper is to investigate the impact of structural breaks on volatility and Value-at-Risk (henceforth ‘VAR’) forecasting. In the financial sector, VAR has become a widely used tool for measuring risk. As a risk measure, it is used in setting capital requirements, trading limits, making portfolio decisions and evaluating investment performance. It is also accepted as a risk measure by regulation. There is a growing body of literature, which, on the one hand, focuses on the theoretical and empirical properties of VAR as a risk measure, and, on the other, the performance of various quantitative methods available for estimation. This paper joins the latter branch of this literature by comparing different methods for forecasting volatility, where forecasting performance is evaluated according to both statistical and economic criterion—i.e. how good these methods are at forecasting VAR. The rationale behind the use of different criteria is the following. First, volatility is not observable, but the standard approximation (squared return) used is very noisy. Furthermore, volatility is forecasted in order to use it as an input to price assets, measure the risk of investment alternatives or calculate VAR and capital. This raises the obvious requirement of assessing our forecast in a practical framework. In this regard, a VAR forecast is considered accurate if the number of hits (cases, when the actual loss exceeds the value of VAR) corresponds to the chosen coverage rate or probability level. Furthermore, we require the failures to be independent through time, as any clustering of VAR failures could easily force a bank into bankruptcy. The presence of structural breaks (henceforth ‘SB’) in economic and financial time series has long been suspected. The last decade has seen important advances in the econometrics of detecting shifts of unknown number and location in more and more general frameworks. Armed with these new techniques, long-standing issues have been investigated, often yielding new results and insights in areas such as efficient market hypothesis, existence of cointegrating relationship, unit root tests or volatility modelling. Regarding financial time series, ample of evidence of the presence of SBs has been found. Moreover, SBs are also shown to contribute to the fatness of the tail of the asset return’s distribution, which is the major challenge in modelling return and in particular in calculating Value-at-Risk. Should the return distribution be leptokurtic, the often-employed assumption of normality renders the VAR forecast inaccurate and results in lower-than-necessary capital. These findings provide the motivation for undertaking the analysis outlined above.

The approach followed in this paper is unique, in the sense that models with and without
structural breaks are compared in terms of their VAR forecast. As far as I know no similar attempt has been taken in the literature. I use structural break tests (developed by Bai and Perron (1998, 2003)) to detect the presence of and to estimate the location of breaks. Then, the VAR forecast of different unconditional and conditional models (MA, EWMA, AR-GARCH, AR-GARCH with structural break dummies) are compared. When the breaks are ignored, the models can be misspecified and their forecasting performance deteriorates. However, the model with the best fit does not necessarily provide better forecasts. Thus, the major issue is whether a model of better fit leads to superior out-of-sample forecasts.

Our findings may have implications in all areas where VAR is used as a risk measure by financial institutions, mainly in financial risk management, but also in pricing and in performance evaluation. For the central bank and the regulator, the major concern is how well VAR models perform to set capital. In addition, due to better modelling of volatility, the paper can contribute to the methodology of constructing stress scenarios, where we use VAR-type measures as well.

We perform the analysis on daily log-returns of the Hungarian stock index (BUX) for the period 1995-2002. Emerging markets are especially prone to regime changes, therefore they provide excellent examples for investigating the problem of SBs.

The structure of the paper is as follows: First, the basic concepts (VAR and SB) are clarified along with a review of the empirical literature. Then, the methodology of the paper is outlined. The third and fourth chapters report the results of the SB tests and the findings based on the evaluation of forecasting performance. Finally, the conclusions are presented.
I. Structural breaks and VAR

In this chapter the two major concepts of the paper are discussed along with the relevant literature.

1. VAR

During the 1990s Value-at-Risk \(^1\) became a standard measure of risk in financial decisions. It replaced standard deviation or volatility in several areas, such as investment decisions (Sharpe ratio based on VAR) and financial risk management. Why it became so popular, what are the major critiques, what are the empirical findings — those issues are investigated in the next chapter.

1.1. The concept of VAR

VAR is an estimate of the maximum loss one might incur on a portfolio — i.e. a change in its value relative to its current value or to a benchmark — with a given probability during a given holding period. VAR can be interpreted as the amount of capital needed to avoid default with a given probability.

Let \( X \) be a random variable (e.g. return). \(^2\) The VAR is an (\( \alpha \)) quantile measure of risk, \( \text{VAR}_\alpha (X) \), where \( \text{Prob}(X<\text{VAR}) = \alpha \). Or given the cumulative distribution function \( F \) of \( X \), \( \text{VAR}_\alpha (X) = F^{-1}(\alpha) \), where \( \alpha \) is the chosen probability level.

VAR is not a simple number: it is conditional on several parameters. We have to choose the probability level, which is typically 95-98% for financial institutions, although 99% is required by regulators. This probability should correspond to the risk appetite of the owners of the bank. \(^3\) The high confidence level required by regulation reflects its conservative nature. The decision on the holding period depends on the type of asset/investment and the liquidity of the market. The holding period should correspond to the time needed to liquidate the position. Banks usually calculate daily VARs, which are then aggregated into 10-day VAR (for regulatory reporting) by using the square root of time rule. As for the time window over which parameters are estimated, there are two conflicting considerations. One is the need to rely more recent observations, the other to

\(^1\) For more on VAR, see Jorion (2002), Duffies and Pan (1997).

\(^2\) VAR can be expressed in terms of prices but also as returns.

\(^3\) Banks can derive the confidence level from the default data of rating agencies, by simply looking up the default probabilities of the desired credit rating. However, the degree of risk aversion or risk appetite is only partially captured by the choice of probability level.
obtain reliable estimation, which requires a longer time window. The Basel Regulation recommends at least one-year estimation window.

During the 1990s **VAR** (often calculated from internal risk models) became the common standard market risk measure, used by financial institutions. Aside from its support by regulation, why has it become so popular? Instead of concentrating on individual sources of risk and certain types of asset (as previous risk management techniques), VAR is able to capture the **overall risk** of the entire portfolio. In doing so, VAR takes into account the interaction between elements of the portfolio (correlation), and thus captures the effect of diversification and hedging on the overall portfolio risk. It calculates not only the sensitivity of asset returns to risk factors, but also **calculates the associated losses**, taking into account current knowledge on the behaviour of risk factors. As risk is expressed in terms of losses, any type of risk can be compared or added. As we cannot know the future with certainty, VAR should be a probability risk measure. To provide a probability measure, internal VAR models rely extensively on quantitative models and modern statistical techniques.

VAR has not only become a standard risk measure, but is also employed as an **active risk management tool**. At strategic level, it is used to measure risk and to make decision on hedging accordingly, but it also provides tools to set capital. At trading desk level trading limits, capital allocation, performance measurement (RAROC) are all based on VAR.

**I.1.2. Critique of VAR**

Despite its above-mentioned appeal, VAR is often criticised on several grounds. First, VAR methods are criticised because of their unrealistic assumptions and simplification (normality and linearity). Second, VAR estimates are not robust—they are very sensitive to the choice on parameters (holding period, probability) and valuation models, as well as the composition of the portfolio. Beyond model/parameter risk, there is also significant implementation risk. The forecasting properties of certain VAR models can be quite weak even during normal periods. Finally, the strongest critique of VAR is its collapse during crises, when it is most needed.⁴ During a crisis volatility jumps, correlations breakdown, the relationship between market and credit risk changes and liquidity evaporates.

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⁴Some even argue that VAR played a role in amplifying the impact of crises in 1997-1998. See Dunbar (2001), Persaud (2000). There is also a growing body of literature on the impact of VAR constraints on investment decisions and price dynamics.
The theoretical critique focuses on the limitations of VAR as a risk measure. The first problem is that VAR is not a coherent risk measure (see Altzn er et al.). Unless multivariate normality is assumed, VAR fails to be sub-additive. One can easily construct a portfolio (typically of credits or options) and divide it into sub-portfolios in such a way, that the VAR of the entire portfolio exceeds the sum of calculated sub-VARs. The lack of sub-additivity limits the applicability of VAR both in regulation and risk management. It creates an incentive against diversification and fails to recognise the concentration of risk. Furthermore, it renders the allocation of capital and trading limits difficult. It also encourages firms to organise their activities in separate entities—in order to avoid a higher capital requirement.

VAR and VAR-based risk management is also criticised for focusing on the probability of loss, but ignoring its magnitude. However, risk-averse investors are not only interested in the probability of loss, but also in the expected value of losses below VAR. Consequently, VAR cannot take into account differences in risk attitude (see Schroder, 2000) and leads to sub-optimal decisions for risk-averse agents. In the special case of normally distributed asset returns, the use of VAR corresponds to the Markovitz \( \mu/\sigma \) criterion and is capable of comparing the risk of alternative portfolios. The Sharpe ratio \((\mu-f_\mu)/\text{VAR}\) used in optimal capital allocation and the RAROC measures \(\mu/\text{VAR}\) used in performance measurement are similarly appropriate only if normality of asset returns applies. However, in a non-normal world, the above-described VAR-based measures result in sub-optimal decisions. For example, a risk-averse investor might choose A instead of B, according to their VARs, although B has much fatter tail beyond VAR. Similarly, when used as a trading limit or part of a compensation scheme, VAR creates an incentive for traders to choose “high loss with low probability” strategies. Which generate almost sure profit but large losses (below VAR) with a small probability.

To address these shortcomings, Schroder recommends a measure of shortfall risk, which is the generalised concept of VAR and suitable for any distribution and any kind of risk-averse investors. The so-called lower partial moment is defined as:

\[
lpm_n(z) = \int_{-\infty}^{z} (z-R)^n dF(R). \]

Where \( z \) is the minimum return and \( F \) is the return’s distribution. The special case of \( n=0 \) gives VAR itself. When \( n=1 \), it is called the **target shortfall**, that is the expected loss below \( z \). According to Schroder, the shape of the utility function, in particular the degree of risk aversion should decide on \( n \) and \( z \). A risk-

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\(^5\) We chose \( \alpha \), then solving \( lpm(z)=\alpha \) we get \( z=\text{VAR}_\alpha \).
averse investor has a utility function with $U'>0$ and $U''<0$, in which case she should use $n=1$. If she is even more risk averse ($U'''>0$) then $n=2$ should be used. That means the further away the return is from the minimum $(\bar{x})$, the larger the weight attached to it.

In order to address these shortcomings, in addition to VAR, this paper investigates a modified version of Target (or Expected) Shortfall:

$$EES_\alpha(X) = E[X - VAR_\alpha(X) | X \geq VAR_\alpha(X)].$$

Here, instead of using the expected value of shortfall, the expected value of “excess” shortfall (shortfall – VAR) is calculated.

### I.1.3. Empirical literature on VAR

Here only a very small—but for us the most relevant—segment of the empirical literature is reviewed: those papers which compare different volatility forecasting methods in terms of their VAR forecasts. The range of models to be compared covers historical MA, EWMA, GARCH, Stochastic Volatility (SV) and implied volatility (IV) models.

Overall, moving average models seem to be the worst. Otherwise, there is no straightforward result, and one cannot establish a ranking among the models. The results are very sensitive to the type of loss functions used, the chosen probability level of VAR, the period being turbulent or normal etc. Some also find a trade-off between model sophistication and uncertainty. To illustrate the above findings, for example Lehar (2002) finds that more complex volatility models (GARCH and SV) are unable to improve on constant volatility models for VAR forecast, although they do for option pricing. In contrast, in Gonzalez et al. (2002), SV is the preferred model to forecast VAR of the S&P500. Wong et al. (2003) concludes that GARCH models, often found superior in forecasting volatility, consistently fail the Basel backtest. Several papers investigate the issue of trade-off in model choice; for example Caporin (2003) finds that the EWMA (used by the popular RiskMetrics approach) compared to GARCH-based VAR forecast provides the best efficiency at a lower level of complexity. Bams (2002) draws similar conclusions, although sophisticated tail modelling results in better VAR estimates but with more uncertainty. EWMA is a special case of GARCH. Supposing that GDP is close to be integrated, the use of the more general GARCH model introduces estimation error, which might result in the superiority of EWMA. Guermat and Harris (2002) show that EWMA-based VAR forecasts are excessively volatile and unnecessarily high, when returns do not have conditionally normal distribution but fat
tail. This is because EWMA put too much weight on extremes. According to **Brooks and Persand (2003)**, the relative performance of different models depends on the loss function used. However, GARCH models provide reasonably accurate VAR. **Christoffersen (2001)** shows that different models (EWMA, GARCH, Implied Volatility) might be optimal for different probability levels. Finally in the paper by **Billio (2000)** regime-switching models outperform EWMA and GARCH VAR forecasts for Italian stocks. Actually that is one of the two papers, which investigates an issue similar to that of our paper. The other is **Guidolin and Timmermann (2003)**, who find regime switching models’ out of sample risk forecast superior at longer horizon (between 6 month – 2 years) only. They investigate a portfolio of US stocks, bonds and T-bills.

In my view, SB tests and the use of dummmies result in simpler and less parameterised models compared to regime switching models.

**I.2. Structural breaks**

The majority of quantitative methods, econometric models applied in risk management assume the stability of individual return (mean and volatility) processes.\(^8\) However, the longer the data period used to estimate the data generating process (DGP) the more likely, that structural breaks occur.

**I.2.1. What are structural breaks?**

The simplest definition of SB is an instability or break in the parameters of the data generating process (or in the forecasting model). Let us take a simple example. Suppose the return series is described as an AR(1) process.

\[
y_t = \mu + \beta \ast y_{t-1} + \varepsilon_t
\]

\[
E[y] = \frac{\mu}{1 - \beta}
\]

\[
Var[y] = \frac{\sigma^2}{1 - \beta^2}
\]

where \(\varepsilon_t \sim iid(0, \sigma^2)\). SB can occur due to a change in the intercept (level), in the slope parameter or in the volatility of the error term. Regarding its moments, the first two causes its mean to change, while the second and third cause its volatility to change. That is, a change in the intercept/residual’s volatility does not have any effect on the variance/mean, since they do not depend on those parameters. A change in the slope parameter influences both moments. SBs are detected by finding changes in the

\(^8\) SB can appear not only in time series but also in structural models. However, this aspect is not covered in detail in this paper.
parameters of the conditional model, but they imply shifts in the unconditional mean and/or variance. Although the DGP is non-stationary over the entire period, it may be stationary between two consecutive SBs.\(^9\)

To obtain a more precise definition, it is worthwhile to highlight the difference between SB and other concepts. According to Timmermann, what distinguishes a break from a \textbf{shock} is that, although both are low frequency events, the former has a long-run (persistent) effect. Let us take an example from Hungarian exchange rate history. While the official devaluations of the currency during the narrow intervention band regime can be considered as outliers, the increased volatility following the widening of intervention band suggests a SB.

It is also worth mentioning here the importance and difficulty in differentiating between the time-varying nature of conditional variance (called \textbf{heteroscedasticity}) and shifts in unconditional variance. While the first describes the short-term dynamics of volatility, the second refers to change in the unconditional volatility.

Brooks (2002) define \textbf{SBs versus regime switches} as irreversible change (once and for all) as opposed to moving and then reverting from one regime to the other. This seems to be a straightforward differentiation. In practice, however, it is not always obvious even \textit{ex post}, whether it was a SB or regime shift. Moreover, the ways we can deal and test them is not so distinct either. For example, regime switching models may include a non-recurring state, which is well suited to model SBs (see Timmermann, 2001). On the other hand, SB tests are capable of detecting regime shifts of a recurring nature.\(^10\)

Why do structural breaks occur? They are usually associated with significant economic and political events. Changes in foreign exchange regimes (from fixed to free or managed float), monetary policy shifts, other economic policy measures (liberalisation of capital movements), building up and bursting of asset price bubbles and even the development of stock markets (in terms of efficiency), or shifts in the required risk premium. Most of the literature on SB tests focuses on one-off shifts (SB) in mean and volatility. There is, however, no guarantee that breaks detected by the tests are of this type. In some cases the underlying event is one-off type (capital market liberalisation or monetary policy regime changes). In other cases changes are related to the business cycle, which can be

\(^9\) In a more general univariate setting, where a trend component is included too, changes can take place not only in level and volatility, but also in the trend (characterises many macroeconomic and financial series, see for example, Wang and Zivot).

\(^{10}\) This idea is utilised in Gonzalo and Pitarakis (2002), who uses SB tests to determine the number of regime switches in threshold models.
rather captured by regime switching models, or to moves which are not easily identifiable. Sometimes pure luck causes lower level of volatility: a decreased volatility of exogenous shocks. Here, we take the practical view: find any break and include them in the model: irrespective of whether the underlying event is a type of the first or the second, or whether the causes are identified or not. Nevertheless, we do seek for plausible explanation for each break that is found.

I.2.2. Why do structural breaks matter?

There is ample empirical evidence on structural breaks in economic and financial time series (see for example Hansen (2001), Stock and Watson (2002) or Aggarwal et al. (2001)). What problems do unaccounted SBs cause? In general, ignoring the presence of SBs leads to incorrect conclusions regarding the behaviour of certain variables. It has serious implications. To mention only a few:

- When one estimates a model over a period when a structural break occurred (broken trend, for example), the estimated DGP might be misspecified – either its functional form is wrong, or the estimated parameters are far from the true parameters. We not only obtain an incorrect picture of the behaviour of the given variable, but also our forecasts may become unreliable. Clement and Hendry (1998) view SB as a key determinant of forecasting performance. However, being misspecified does not always imply worse forecasts.

- Econometric tests might lose their power and lead to incorrect inferences. SBs cause unit root tests to have low power – we might wrongly fail to reject it. Furthermore, SB can also cause the spurious rejection of unit root. That is, not only is the power low (the probability of not rejecting a wrong $H_0$ is high) but at the same time the 1st type error is high (the probability of wrongly rejecting a true $H_0$). The seminal paper of Perron (1989) showed first how to modify the null and alternative hypothesis of unit root test to incorporate potential breaks in time trend. In contrast to previous empirical results, the application of his new test suggests that many macroeconomic time series may be stationary around a broken deterministic trend.

- In volatility forecasting omitted breaks induce significant bias into conventional volatility estimation (expanding or fixed window moving averages). Pesaran and Timmermann (1999, 2003) devoted two papers to the issue of how SBs influence the decision on optimal window size in forecasting and on model selection with the aim of sign forecasting.

- The presence of breaks also influences the estimation of other parameters, when more
complicated volatility models (GARCH, ARFIMA) are used. For example, omitted breaks cause an increase in the estimated persistence and the order of integration in GARCH/ARFIMA models. It has consequences on risk management, asset allocation, asset pricing—in all areas where volatility is used.\footnote{We return to this issue in the next section.} One example is forex volatility, which is often found close to be integrated. This persistence disappears or decreases significantly when breaks are taken into account, for example, by simply estimating the models for the sub-periods. If IGARCH is not proper any more, that questions the use of EWMA as well.

- Ordinary descriptive statistics of actual data become misleading when SBs are present: mean, variance, autocorrelation. The verification of any theory which requires comparison of the data generated (simulated) by a model (representing the theory) and the actual data (characterised by its moments) over a period with a SB will lead to incorrect conclusions.

- Often provides different or new results compared to previous studies on issues such as: Efficient Market Hypothesis – random walk versus mean reversion, existence of cointegrating relationships, Purchasing Power Parity, business cycle theory, equity risk premium puzzle, sources of stock return predictability. For example Chaudhuri and Wu (2003) show that accounting for SBs—arising due to liberalisation—in emerging market stock indices leads to the rejection of random walk for many of those markets. This stands in contrast to the previous (rather surprising) evidence. Another area of application of unit root tests adjusted with SB is Purchasing Power Parity tests. Baum et al. (1999) find no evidence on absolute long-run PPP, where PPP corresponds to unit root test in real exchange rates. On the contrary, Sabate et al. (2003) do find evidence on PPP for peseta-sterling when SBs are considered. Cointegration tests were also modified to adjust for SB. Voronkova (2003) shows that this test finds more cointegrating relationship between some Central-European and more mature stock markets than its counterpart without SB.
I.2.3. Connection between SB and other stylised facts of returns

There are some empirical properties of asset returns, which have been challenging the econometric society for a long time. Those stylised facts are: conditional heteroscedasticity and volatility clustering, leverage, fat tail, persistence and long memory. They all characterise the dispersion or the volatility of the return and its dynamics. Fat tail is a distributional property. It means that the number of extreme return observations exceeds that implied by the often-assumed normal distribution. All the others represent facts about the dynamics of volatility. Heteroscedasticity only means time-varying conditional volatility. Leverage is the asymmetric response of volatility to bad versus good news. The remaining concepts have very similar meaning. Volatility clustering is the observed autocorrelation in the time-varying conditional return volatility. It is used to capture the tendency of large/small price changes to be followed by large/small moves. Persistence means that any shock/news has a long-lasting effect on volatility. Long memory is the slowly decaying autocorrelation observed in the absolute return series. As we will see later, these concepts are interrelated. Furthermore, SB may cause all those stylised facts and models with SB provide a promising way to capture them. Volatility clustering and leverage are typically captured by various GARCH models. The GARCH effect does cause fattening of the tail as well. However, accounting for the GARCH effect is often not enough to get normal errors: the standardised error still has higher than normal kurtosis. This calls for additional approaches. Long memory is often modelled by fractionally integrated (ARFIMA) model in absolute return.

How does SB fit into this picture? There is ample evidence based on analytical calculation, simulation and actual data that SBs, on one hand, can generate all the stylised facts above (fat tail, GARCH and long memory). On the other hand, SB and GARCH, SB and long memory are interrelated. They are very easy to confuse. Should any one be present, the estimation of the other is distorted. For example, when either GARCH or long memory is present, SBs are spuriously detected. And vice versa, when SBs are ignored, the long memory parameter and the degree of persistence are overestimated. The connection between SB and long memory and persistence is investigated in several papers. Lamoureux and Lastrapes (1990) were among the first to suggest that the persistence in volatility might be overstated due to structural changes in variance. Granger and Hyung (1999) investigate the relationship between SB and long memory.
They show via simulation that linear processes with SBs in mean can produce the same pattern in sample autocorrelation as fractionally integrated I(d) processes. They also find that the number of breaks is often spuriously overstated when the underlying process is not stationary but I(1) or I(d). There is a positive relationship between d and the number of breaks detected in finite samples (and also the magnitude of break). Even if the I(d) or I(1) underlying process does not have shift, breaks will be found. Their conclusion based on the empirical analysis of the S&P500 is that the choice between I(d) and SB is not obvious, although they prefer the latter one.

Diebold and Inoue (2001) show analytically that long memory and regime switching or structural change can be easily confused under certain conditions (when the number of regime switches is small). D&I argue, that even if structural changes do occur, long memory models may provide a convenient way to generate the observed features of the data and to forecast. I disagree with the latter claim. Long memory and SB do generate similar acf pattern, but they assume different behaviour (stationarity, speed of mean reversion, impact of shocks). This might imply different forecasts, too.

As to empirical applications, Gil-Alana (2002) show, that when mean shift is included in the regression model of US interest rates, the order of integration decreases—still non-stationary, but the mean reversion property accelerates. Gadea et al. (2004) show using the example of three countries’ inflation series that ignoring SBs induces upward bias into the estimation of long-memory parameters (in the period of 1874-1998 for UK, Italy and Spain). Bachmann and Dubois (2001) highlight the difficulties of trying to disentangle unconditional breaks and conditional heteroscedasticity. Applying their alternative methods12 they detect much fewer breaks in variance in 10 emerging stock market indices than previous studies. Morana et al. (2002) investigates fx rates and show that long memory is only partially explained by unaccounted regime changes. Regarding forecasting performance Markov switching models of realised volatility outperform ARFIMA models only for longer horizon (5-10 days) but not for 1-day forecasts.

Based on the above evidence, the inclusion of SB in volatility models seems to provide a very promising way to capture the leptokurtosis present in stock returns and avoid some spurious findings on the behaviour of return. It is of special importance when the aim is to forecast VAR, for example with the purpose of setting capital, where the tail is in the centre of interest.

12 They apply ISCC on the aggregated time series where heteroscedasticity disappears.
I.2.4. Simulation

To illustrate the impact of structural breaks discussed in the previous chapter, a simple simulation is conducted. Based on the actual estimation on unconditional variances in our dataset (see below), 8 samples of normally distributed returns are generated (N=5000) with zero mean and the following variances:

1. Table: Variances used in the simulation exercise

<table>
<thead>
<tr>
<th></th>
<th>x1</th>
<th>x2</th>
<th>x3</th>
<th>x4</th>
<th>x5</th>
<th>x6</th>
<th>x7</th>
<th>x8</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.5</td>
<td>1</td>
<td>3</td>
<td>5</td>
<td>7</td>
<td>10</td>
<td>15</td>
<td>20</td>
</tr>
</tbody>
</table>

Then structural breaks are induced by taking all possible combinations of the above variables. For example, x₁ followed by x₂ gives the first series (out of altogether 28). The series with structural breaks are then investigated. Although the series is normally distributed between and after the SB, the entire series is found to be leptokurtic. The larger the break is, the fatter the tail is.

2. Table: Kurtosis in various models

<table>
<thead>
<tr>
<th></th>
<th>x1</th>
<th>x2</th>
<th>x3</th>
<th>x4</th>
<th>x5</th>
<th>x6</th>
<th>x7</th>
<th>x8</th>
</tr>
</thead>
<tbody>
<tr>
<td>x1</td>
<td>3.381</td>
<td>3.736</td>
<td>3.220</td>
<td>3.051</td>
<td>3.141</td>
<td>3.127</td>
<td></td>
<td></td>
</tr>
<tr>
<td>x2</td>
<td>4.539</td>
<td>4.374</td>
<td>3.410</td>
<td>3.051</td>
<td>3.141</td>
<td>3.127</td>
<td></td>
<td></td>
</tr>
<tr>
<td>x3</td>
<td>5.080</td>
<td>4.582</td>
<td>3.928</td>
<td>3.376</td>
<td>3.344</td>
<td>3.127</td>
<td></td>
<td></td>
</tr>
<tr>
<td>x4</td>
<td>5.173</td>
<td>5.122</td>
<td>4.226</td>
<td>3.770</td>
<td>3.442</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x5</td>
<td>5.624</td>
<td>5.122</td>
<td>4.226</td>
<td>3.127</td>
<td>3.442</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x6</td>
<td>5.506</td>
<td>5.164</td>
<td>4.692</td>
<td>3.442</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x7</td>
<td>5.775</td>
<td>5.506</td>
<td>4.692</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x8</td>
<td>5.775</td>
<td>5.506</td>
<td></td>
<td></td>
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</tbody>
</table>

The autocorrelation function on the squared return series shows significant ARCH effect for all of the series, recommending the use of GARCH models. Although we know that in the two sub-periods the series is iid, without any ARCH effect. LM-ARCH tests give the same conclusion. When GARCH models are estimated, the parameters are significant and in all cases the persistence is very high (close to 1).

This simple exercise demonstrates how SBs can lead to spurious results – strong ARCH effect and very strong persistence is found by the standard tests when none is present.

I.2.5. Different options to model shifts in the DGP

There are different approaches to model changes in the DGP depending on the nature of the shift. One is regime-switching model. Typically, two-three (finite number of) regimes are defined, one representing normal or quiet markets, the other more turbulent...
periods of high volatility. So-called Markov switching models define a probability transition matrix, which governs the shifts between regimes. They can capture for example business cycle effects. One of the major challenges of MSMs is the estimation of transition probabilities simultaneously with the parameters of each regime.

In threshold auto-regressive models the value of a state variable governs movements between regimes, that is the major difference between threshold and MSM models. When it exceeds a threshold, the DGP jumps from one regime into the other, whereas in MSM the transition probabilities determine the dynamics of the process.

Mixture of normal distributions is another option to capture the changes of return distribution. Regime switching represents one type of this. However, mixtures of normals provide much simpler models too, when shifts are allowed only in the variance but not in the mean. In that case, one can assume that the sample is drawn from two distributions with the same mean but with different volatility. Mixtures of normals are often successful in mimicking the observed high kurtosis of actual data (see Alexander (2002)).

Another alternative method is extreme value theory, which concentrates on extreme observations only. The implicit assumption of EVT is that extreme observations are generated by a different DGP.

SB models deal with shifts in a specific way. The so-called two-step approaches use specific tests designed to detect SBs endogenously, then incorporate them into the model (by slope and/or intercept dummies). Others simply adjust the estimation window; ignore all or some of the pre-break observations. In this paper, a two-step approach is followed.

I.2.6. Tests to detect the number and location of structural breaks

Recent econometric advances provide a rich methodology to test endogenously the presence of SBs of unknown location in a more and more general framework. Algorithms and tests have been developed to detect and locate breaks. The econometrics of SBs concentrates on the following issues:
- Construct tests to detect the presence and number of break.
- Build an algorithm to find the breaks sequentially or simultaneously;

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13 Sometimes three or more states are defined.

14 Alternatives to SB tests are the fluctuation test of Sen, Nyblom instability test, graphical analysis of recursive and rolling window parameter estimates, tests based on recursive estimates etc.

15 Where the null hypothesis against the alternative is 1 break against 0, k against 0, or k+1 against k.
- Establish the asymptotic properties (convergence, consistency, distribution) of the location estimator. Then calculate confidence intervals around them.

There are three basic approaches followed in the construction of SB tests: OLS-based tests, CUSUM-type tests and SB detection as a model selection exercise (based on Information Criterion).

**OLS-based tests** are used to detect breaks both in mean and variance. They are OLS-based because the derived test statistics rely on the sum of squared of OLS residuals. The roots of SB tests go back to the Chow-test. When the **time of the break is known**, one can use Chow (F-) test (when errors are normal) or the Wald, Lagrange Multiplier or Likelihood Ratio statistics (when errors are not normal) to test the null of no break. They are based on the comparison of the SSR of a restricted (no break) and un-restricted (with break) model.

When the **k is not known**, it complicates the testing procedure because of the presence of a nuisance parameter (k), which appears only in the alternative hypothesis. The OLS-based test for the case of unknown break location (k) was first developed by Quandt (1960). He came up with the idea to calculate the Wald statistic over all possible values of k,\(^{16}\) then to take the maximum value of the Wald statistics, which is called the SupWald. However, the limiting distribution is not \(\chi^2\) any more – unlike in the case of known k. Years after the idea of Quandt, Andrews (1993) derived the limiting distribution of the SupWald statistic and critical values. The p-values were tabulated later by Hansen (1997). When the null is rejected, the location of break (k) is estimated as the argmax SupWald.\(^{17}\)

OLS-based SB test are used to detect breaks in mean and variance as well. Later, other versions of the test (m against 0, m+1 against m breaks) were developed.

To find more than 1 break the testing procedure should be applied sequentially. When the first break is found, one should rerun the test on the sub-samples, as long as the SB test statistic is significant. Often the found break points are refined. Let k1 and k2 be the first two break points found, then k1 needs to be re-estimated on the sub-period \([1,k2]\) or \([k2,T]\) depending on whether k1<k2 or k1>k2.

An alternative procedure for testing shifts in the **variance** of a series was introduced by Inclan and Tiao (1994). It goes back to the **CUSUM** test of Brown, Durbin and Evans (1975), which was originally formulated to test the stability of a forecasting model. The

\(^{16}\) More precisely the beginning and end of the sample is trimmed, to ensure large enough sub-samples to estimate parameters. The typical value of trimming is between 5% and 15%.

\(^{17}\) Later, Andrews developed the Exponential and Average - instead of Sup - version of the statistic (see Andrews and Ploberger (1994)).
test statistic of Inclan and Tiao is based on the centred and normalised cumulative sum of squares of (the standardised or demeaned) return, which should oscillate around zero. The algorithm (called ICSS – iterated cumulative sum of squares) to find all the breaks, one at a time, is brought about as above: carry on locating breaks until the test statistic becomes insignificant. The major advantage of their approach lies in the computational simplicity relative to other approaches. One of its weaknesses is the iid assumption, which is not met when the volatility follows a GARCH process.

Finally, the decision on the number and location of breaks can be interpreted as a model selection problem. Choose the model with the minimum value of Information Criterion (AIC and BIC are used). If for any k the model with structural break has a lower IC value than the model without SB, it is regarded as evidence of a break. When the first break is found, one can proceed in applying the idea for the sub-periods.

Empirical papers very often use more than one approach. It seems that different approaches may yield different results (number or location of breaks). Therefore, in this paper two approaches are used – OLS and IC-based.

**II. Methodology of the paper**

**II.1. Description of the data**

Trading on the Budapest Stock Exchange (BSE) started in 1990. A real boom in equity capitalisation and trading volume started in 1996. It followed a period of consolidation and privatisation, which was a sharp response to the transition crisis. The Hungarian economy and the equity section of BSE have some characteristics, which may be important when the results of our analysis are assessed. Hungary is a small open market economy with entirely liberalised capital markets. The share of foreign investors is high in the domestic capital markets, amounting to around 70% on the equity section of BSE throughout the period investigated. Furthermore, Hungary is an emerging country. Therefore, prices (and volatility) on the stock market are not only affected by fundamentals and profit prospects of registered companies, but also by the international economic environment, the risk appetite of international investors, and the overall assessment of emerging market risk. The fact that BSE is integrated into global markets is documented in several papers. Voronkova (2003) find evidence on long-term linkages among three Central European stock markets (Hungary, Poland and Czech Republic) and between Hungary and Germany, France and the USA. Scheicher (2001)

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18 The same figure is about 30% for Poland.
also reported strong influence of global factors on the BUX. Compared to other CEE stock markets, the **liquidity** on BSE is relatively high (see Jaksity 2002), but much smaller than on more developed markets. Therefore, foreign capital in- and outflows can cause jumps in prices on the market.

The following table documents the rapid development of the equity section on BSE, in terms of capitalisation and turnover, which halted significantly only in 2001, due to the general recession on capital markets.¹⁹

### Table. Some indicators of the equity section on BSE

<table>
<thead>
<tr>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of equities listed</td>
<td>42</td>
<td>45</td>
<td>49</td>
<td>55</td>
<td>66</td>
<td>60</td>
<td>56</td>
<td>49</td>
</tr>
<tr>
<td>Capitalisation as % of GDP</td>
<td>5.99</td>
<td>12.86</td>
<td>36.64</td>
<td>29.9</td>
<td>36.05</td>
<td>20.25</td>
<td>19.38</td>
<td>19.4</td>
</tr>
<tr>
<td>Capitalisation (bn HUF)</td>
<td>327.8</td>
<td>852.5</td>
<td>3058.4</td>
<td>3020.1</td>
<td>4144.9</td>
<td>3393.9</td>
<td>2848.8</td>
<td>2947.2</td>
</tr>
<tr>
<td>Turnover (bn HUF, double counted)</td>
<td>87.3</td>
<td>490.5</td>
<td>2872.7</td>
<td>6920.7</td>
<td>6862.7</td>
<td>6834.1</td>
<td>2771.4</td>
<td>3027.4</td>
</tr>
<tr>
<td>Number of transactions</td>
<td>60 851</td>
<td>153 937</td>
<td>478 236</td>
<td>1 011 514</td>
<td>1 461 482</td>
<td>1 612 482</td>
<td>902 381</td>
<td>730 822</td>
</tr>
</tbody>
</table>

The **BUX** was launched on 2 January 1991 with a value of 1000 points. The index took its final form on 1 January 1995, and that is where our time series starts. However, technical details and rules did change even after 1995. For example, prior to April 1997 it was calculated on the basis of daily average prices of the basket stocks. After that date, 5-seconds index prices were calculated, together with the closing prices of the BUX.

Another important change took place in 1998, when open-outcry trading was replaced by electronic remote trading. It is a capitalisation-weighted index reflecting dividend payments. The maximum number of equities allowed in the basket is 25. Prior to October 1999, the weight of each single security was limited to 15%, and since then the index is calculated from capitalisation adjusted by free float.

The evolution of the index value and return of the BUX are displayed on Graph 1 and 2.

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¹⁹ Burst of the dot-com bubble, 9/11.
II.2. Modelling the BUX return with SB

In this paper, the return and volatility of BUX is modelled as an AR-GARCH process with SB dummies. Why may SBs occur? Regarding unconditional mean, the weak form of the efficient market hypothesis implies that the return series is not autocorrelated (the AR term is insignificant). However, in the case of the BUX we assume that during the first part of the observed period the market was not efficient enough to remove all autocorrelation from the return series. It was a newly founded stock market in an emerging country, and consequently initial capitalisation, turnover and liquidity were low. Transaction cost or lack of experience may have played a role as well. Even if we
suppose that markets are efficient, and prices follow a random walk with drift, the drift component might change with the business cycle or the risk premium – which might be captured by changes in the intercept term or the level of long-run mean. In summary, changes in the long-run mean might occur due to: evolving efficiency, changes in risk premium or in the business cycle.

As to volatility, we assume that GARCH representation will be necessary: this is supported by the data as well. Shifts in the level of variance are likely to occur more often than in case of the mean. All of the events affecting the slope parameter of the mean equation will trigger shifts in volatility as well. Therefore, changes in the efficiency of the market do alter the long-run level of volatility as well. Not only the level, but the persistence of volatility is also affected by increasing efficiency (we use efficiency in terms of speed of prices adjusting to new information). In addition, there is ample of empirical evidence on a positive relationship between trading volume and volatility. Thus, the rapid expansion of stock markets in emerging markets might have contradictory impacts on volatility: supposing that some predictability (significant AR term) was present in the series, increasing efficiency tends to lower the level and persistence of volatility, but larger volume might push its level up. Volatility is raised due to other reasons too, for example when jumps/news in the return series arrive more often and are of larger magnitude than usual (shift in the volatility of error term). The increasing integration of the local stock market into international capital markets may amplify that impact.

Following the recommendations of Pitarakis (2002), first we detect breaks in the mean, then in the variance. A break in mean is defined as break in the intercept and the AR term.

The log return series ($y_t = 100*\ln(P_t/P_{t-1})$) is specified as an AR(1)-GARCH(1,1) process:

$$y_t = \mu + \phi * y_{t-1} + u_t$$

$$u_t = \varepsilon_t * h_t$$

$$h_t^2 = \alpha_0 + \alpha_1 * u_{t-1}^2 + \beta * h_{t-1}^2$$

where $\varepsilon_t \sim NID(0,1)$.

We run test on the first equation to check if there was any break in the mean parameters ($\mu, \phi$). When the number $n$ and the location $(k_{m,i}, i=1,…,n)$ of the breaks have been

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20 See, for example, Gallant et al. (1992), Jones et al. (1994). The theoretical support is given by the market microstructure literature, which links trade (order flow) to price adjustment, which causes the positive correlation between volatility and volume. Trading volume is proved to be helpful in
estimated, we assume that both the intercept and slope parameters are affected. Then the mean equation with structural breaks takes the form of:

\[ y_t = \mu + \sum_{j=1}^{n} \mu_j * D_{ij} + \sum_{i=1}^{n+1} \phi_i * D_i * y_{t-1} + u_t \]

where \( D_{ij} = 1 \) for \( k_{m,j-1} \leq t < k_{m,j} \), 0 otherwise; \( k_{m,j} \) is the location of i-th break found in mean and \( k_{m,0} = 1 \) (the first observation).

The model for each (altogether \( n+1 \)) sub-period is given by the above estimated parameters. However, while \( \phi_i \) equals the slope for each period between two consecutive breaks, the \( \mu_i \) gives the deviation of intercept in the i-th sub-period from that in the last, \((n+1)\text{st}\) period.\(^{21}\) We use the above models irrespective of the significance of individual parameters.

In order to run SB tests in variance, we take the squared errors of the above equation \((u_t^2)\), and use the Bai-Perron algorithm again. We are looking for break in the mean of the variance.\(^{22}\)

\[ u_t^2 = v + \sum_{i=1}^{m} v_i * D_i, \quad \text{where } D_i = 1 \quad \text{for } k_{v,i-1} \leq t < k_{v,i} \]

We test the null of \( v_i = 0 \) against the non-linearity of parameters. After \( m \) and \( k_i \)'s, the number and location of breaks in variance have been identified, the following AR-GARCH specification is estimated by quasi ML method.

\[ y_t = \mu + \sum_{j=1}^{n} \mu_j * D_{ij} + \sum_{i=1}^{n+1} \phi_i * D_i * y_{t-1} + u_t \]

\[ u_t = \epsilon_t * h_t \]

\[ h_t^2 = \alpha_0 + \sum_{j=1}^{m} \alpha_{0,j} * D_j + \alpha_1 * u_{t-1}^2 + \beta * h_{t-1}^2 \]

II.3. The Bai-Perron method to detect SBs

Bai and Perron (1998 and 2003) consider a linear regression model with multiple but unknown numbers of structural changes in a very general setup. Their methods can be applied for both pure and partial structural breaks. They allow serial correlation and explaining heteroscedasticity as well– see Lamoureux and Lastrapes (1990) and Wagner and Marsh (2004).

\(^{21}\) To avoid multicollinearity only n intercept dummy can be introduced.

\(^{22}\) That means, even when it is a GARCH process, for the purpose of testing SB, we substitute it by a simple constant mean. This kind of model misspecification does not influence our result. The tests have power against this type of misspecification.
heteroscedasticity of errors, different distribution of regressors and errors across segments/sub-periods.

First let us consider the **tests used to detect the presence of SBs**. In addition to the standard SupWald test of 1 break against 0, they also propose test with an alternative hypothesis of unknown number of SBs up to a maximum, and \(l+1\) number of changes against 1.

To **decide the number and location** of SBs their programs include several alternatives. First, they develop an efficient algorithm based on dynamic programming to estimate all the breaks simultaneously, where the number of breaks is fixed. It is called **Global Minimisation** of RSS. The problem with global minimisation is that the RSS continues to decrease with the number of breaks. One usual response to this is to use **Information Criterion** (a penalty factor) to decide the number of changes. Their Gauss program includes an IC based algorithm (a modified Bayesian Information Criterion). However, Bai and Perron recommend a **sequential procedure**. It starts with the first \(k\), obtained from global minimisation and carries on estimating the break points until their new test statistic, \(F(l+1|l)\), becomes insignificant. As they point out, compared to the IC-based approach, the sequential procedure has the advantage of being able to take into account heterogeneity of data across segments and serial correlation of errors. Their simulation results (Perron 1997, Bai and Perron 2000) also highlight the sensitivities of individual methods to different assumptions, and also provide advice on how to improve results. Overall, they find the sequential method performs better than IC-based approaches.

Finally, the break point estimates gained from the sequential method can be improved by applying their **repartition method**. They also construct confidence intervals at 5% under very general assumptions about the data and errors.
II.4. Volatility forecasting models and evaluation criteria

We use the following models to forecast volatility and VAR:

4. Table: Volatility forecasting models to be compared

<table>
<thead>
<tr>
<th>Models</th>
<th>Volatility forecast</th>
<th>VAR forecast</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Unconditional models:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MA250</td>
<td>( \hat{\sigma}<em>{t+1}^2 = \sum</em>{i=t-240}^{t} r_i^2 )</td>
<td>( \text{VAR}<em>{t+1}(\alpha) = F^{-1}(\alpha) \ast \hat{\sigma}</em>{t+1} )</td>
</tr>
<tr>
<td>MA500</td>
<td>( \hat{\sigma}<em>{t+1}^2 = \sum</em>{i=t-499}^{t} r_i^2 )</td>
<td>( \text{VAR}<em>{t+1}(\alpha) = F^{-1}(\alpha) \ast \hat{\sigma}</em>{t+1} )</td>
</tr>
<tr>
<td>EWMA (( \lambda = 0.94 ))</td>
<td>( \hat{\sigma}_{t+1}^2(\lambda) = (1 - \lambda) \ast r_t^2 + \lambda \ast \sigma_t^2(\lambda) )</td>
<td>( \text{VAR}<em>{t+1}(\alpha) = F^{-1}(\alpha) \ast \hat{\sigma}</em>{t+1} )</td>
</tr>
<tr>
<td><strong>Conditional models:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>“Basic” (AR(1)-GARCH(1,1))</td>
<td>( \hat{\sigma}_{t+1}^2 = h_t^2 )</td>
<td>( \text{VAR}<em>{t+k}(\alpha) = \hat{h}</em>{t+k-1} + F^{-1}(\alpha) \ast \hat{\sigma}_{t+k} )</td>
</tr>
<tr>
<td>SB models</td>
<td>( \hat{\sigma}<em>{t+k}^2 = \hat{h}</em>{t+k-1}^2 )</td>
<td>( \text{VAR}<em>{t+k}(\alpha) = \hat{h}</em>{t+k-1} + F^{-1}(\alpha) \ast \hat{\sigma}_{t+k} )</td>
</tr>
</tbody>
</table>

Where:
- \( F^{-1}(\alpha) \) gives the \( \alpha \) quantile of the standard normal distribution (5; 1; 0.5 are used).
- The underlying assumption is that the standardised error of the basic and SB models follow \( \text{N}(0,1) \) distribution.
- \( h_{t+k-1} \) is the conditional variance for \( k=1 \) and the static forecast for \( k>1 \), estimated from the basic and SB models.
- \( \hat{r}_{t+k-1} \) is the fitted value of return for \( k=1 \) and the static forecast for \( k>1 \)
- \( \lambda = 0.94 \) is taken from RiskMetrics

In addition to VAR, we also calculate Expected “Excess” Shortfall as the average of excess losses above VAR.

In case of conditional models we conduct two analyses. First, the entire dataset (1995-2002) is used for estimation and in-sample forecasting is evaluated. Second, we perform a rolling window estimation. We find SBs and estimate the model on the first 500 observations, make 125 (half-year) static forecasts, then roll the estimation window forward, re-estimate the model and do the forecasts again.
Forecasting performance is evaluated by 4 statistical loss functions:

\[ MSE = \frac{1}{T} \sum (\hat{\sigma}_t - \sigma_t)^2; \ MAE = \frac{1}{T} \sum |\hat{\sigma}_t - \sigma_t|; \ AMAPE = \frac{100}{T} \sum \left| \frac{\hat{\sigma}_t - \sigma_t}{\hat{\sigma}_t + \sigma_t} \right|; \]

\[ MAPE = \frac{100}{T} \sum \left| \frac{\hat{\sigma}_t - \sigma_t}{|\sigma_t|} \right|. \]

Here, actual volatility is approximated by the square of demeaned daily return.

When VAR forecasts are assessed, one has to consider the following. VAR is often used to determine the level of capital required both by regulators and the owners of banks. If the actual loss of value of assets is larger than the VAR, it means that the bank does not have enough capital to absorb the losses. Or in other words, compared to its capital, it takes too much risk. On the other hand, if the VAR is much higher on average than the losses which have taken place, the bank holds too much capital, which raises its (opportunity) cost. Consequently, a good VAR forecast is not exceeded much more often than implied by the chosen p. It shouldn’t be too high either, exceeded rarely relative to p. Another issue of concern for both banks and central banks—guarding against systemic risk—is the clustering of hits. Even if a model ensures correct unconditional coverage, the clustering of violations (large losses over VAR coming in clusters) make the VAR model unacceptable, for it increases the probability of bank default.

There is a rich body of literature on how to measure the accuracy of VAR forecasts. In this paper we use the following measures:

- We calculate the number of exceedences (hits) in percentage and compare this to the chosen probability level. We also compare the average value of exceedences in excess of VAR (EES), for we are not only interested in the frequency of model failure but also in the average value of loss over capital.
- We use Kupiec’s unconditional and Christoffersen’s conditional coverage (percentage of failures, hereafter POF) test.

The first one tests the null hypothesis of the actual ratio of failures (when the actual loss exceeds the VAR forecast) being equal the chosen probability level (in our case 5%, 1% and 0.5%). Christoffersen has improved the test by supplementing it with a test of independence. Where the $H_0$ is that the hits are not clustered in time, or in other words, given its capital, the bank could take more risk, thus making more profit.
there is no higher-order dynamics in the hit series. In this sense the test of Christoffersen incorporates both requirements (nominal coverage, independence) argued for above.

Software used: for SB tests: Bai-Perron algorithm in GAUSS (run under Ox), for model estimation and forecasting: PcGive

III. Results over the entire period

III.1. Structural breaks found

First, we apply the BP method on the return series of the BUX to find breaks in mean. One important input parameters users need to decide on in order to run the tests is the minimum distance between two consecutive breaks (h). Since the result is sensitive to this choice, we decided to use several values for h (25, 50, 100). We also run tests and estimate breaks on sub-periods to conduct a kind of sensitivity analysis. Both the results of IC and SP are used. As to significance level, 1% is chosen in the first place, but 5 and 10% results are considered as well.

The tests resulted in a different number of breaks and change-points, depending on different parameters. First, the smaller the fixed length of each segment (h) was, the larger the number of breaks were. However, they occur only in 1997 and 1998, and with decreasing h they became closer to each other. Second, BIC (breaks selected according to Bayesian Information Criterion) and SP (breaks detected by the Sequential Procedure) sometimes give slightly different models.

In light of these findings, we decided to consider more than a single model. To capture the dispersion of results, we included the following models:

---

24 Other tests used in the literature are, for example, the dynamic quantile test developed by Engle and Manganelli (1999) to check non-predictability of hits, or the duration-based test of Christoffersen (2003).

25 Following the recommendation of Mr Pitarakis, we also run the test without imposing heteroscedasticity of error. Mr Pitarakis suggested that some breaks might remain undetected when heteroscedasticity is assumed.

26 The refinement procedure did not change the break-point estimates of the SP.
Then 4 AR-GARCH models with the breakpoints-in-mean above were estimated, the squared errors stored and used to detect **breaks in the volatility**.\(^{27}\) Again three length of segment (h=100, 50, 25) were considered.

As to BIC-based procedure, one striking feature of the results is that given h, the four mean models give almost exactly the same break-point estimates. That implies that the results are robust with respect to different mean model specifications. Furthermore, breaks in variance become closer to each other with h, but their number does not change significantly. There are three periods when breaks in variance occurred. Two coincide with the periods of breaks in mean: in autumn 1997 and 1998. Meaning that in those periods, shifts occurred both in mean and volatility. The additional third is March 1999.

The results got from SP are a bit messier. Compared to the BIC estimates, here we have more and different break-points. Breaks in volatility are found not only in 1997 and 1998, but also at the beginning and at the end of the sample. Nevertheless, given h, the breakpoints of the four mean models are rather similar.

Finally, we decided to choose 2 SB and 2 BIC models, given by h=100 and h=25 for both mean and volatility. Hereafter, these model combinations are referred to as SB1, SB2, SB3 and SB4 (BIC100, BIC25, SP100, SP25). They are AR(1)-GARCH(1,1) models with dummies corresponding to the following break points in mean and volatility:

\[^{27}\text{All four mean models were used to obtain input to estimate breaks by both BIC and SP, irrespective of the mean model relying on SP or BIC-based break-points.}\]
What could be behind the shifts? Most of the breakpoints of mean and volatility are in 1997 and 1998. Hungary was affected by the Russian and to a lesser extent also by the Asian crisis, which occurred in 1997 and 1998. The main channel was the increased risk aversion of international investors due to the events in Russia and Asia. In the former case, Hungary was affected directly too, because of the still significant economic ties between the two countries. However, strong fundamentals ensured a rather quick recovery from both shocks. One might doubt whether or not those events resulted in longer lasting shift in the mean and volatility process. To decide whether or not that was the case or if they only moved temporarily into another regime, I calculated the unconditional mean and volatility for each sub-period between consecutive SBs for the 4 models. They also provide information about the size of the shift.
Different models indicate rather different behaviour of the mean in 1997-1998, but all report a sharp decline in November 1997. They all convey the same message about a real shift after 1998 as well. The unconditional mean declined from roughly 0.2 in 1995-1996 to 0.03 after the turbulent years of 1997-1998. This suggests that either the risk premium declined sharply, or a long-run trend was broken or the market become much more efficient. As to the risk premium, despite the steadily improving sovereign rating of the country (Moody’s rating improved from Ba1 in 1996 to A1 by the end of 2002), the risk premium (spread on government bonds) fluctuated widely, jumping to a higher level after the Russian crisis, falling in 2000 and then rising again after 9/11. Based on this, the role of risk premium in the mean shift can be rejected.

5. Graph: Spread on Hungarian government bonds

Looking at the development of BUX (1.Graph), there is visual evidence of a very
strong upward trend in the value of the index in 1996-1997—resulting in about 100% yearly return, which disappeared after the Asian crisis.\textsuperscript{28} This is very likely to contribute to the decline in the long-run mean. The improving efficiency of the market may have played a role as well. Starting in 1996, the stock market expanded in terms of capitalisation, and turnover, liquidity increased too. The crises are also likely to have a role in the sense, that those events provided a good learning field especially for the domestic agents (brokers, dealers and asset managers).

Regarding volatility, a series of upward shifts can be detected (at the beginning of 1996, end of 1997, 1999).

6. Graph: Unconditional volatility of BUX return implied by various SB models

The increased volatility is likely to be related primarily to the steadily increasing trading volume during this period. During the two crisis periods (Autumn 1997, 1998) volatility jumped dramatically, then it returned back to a level somewhat higher than in 1996. Two forces were at play: increasing efficiency of the market goes in hand with decreased long-run volatility, however the increase in volume\textsuperscript{29} and the two crises might have acted against it. For example, Giot (2003) found evidence of increased volatility, even several years after the Asian crisis, by investigating the VIX and its German counterpart

\textsuperscript{28} According to Ratkovicova (1999), this huge increase is partly attributable to the fact that shares were previously undervalued and foreign investors increased their presence on the market.

\textsuperscript{29} Cunar et al. (2004) investigated SB in volatility of the Spanish stock index over the period 1941-2001. They also find evidence of positive volume-volatility relationship.
Finally, unconditional volatility might increase merely because of the higher frequency or intensity of shocks affecting the market—for example the increased integration of BSE into global markets might have led to greater exposure to external shocks.

The question of SB or regime change cannot be answered without ambiguity—unlike in the case of mean. The jumps in volatility during the crises might be modelled by regime-switching model, however there are also signs of upward shifts in 1996, 1997 and after 1998, which call for modelling with SB.

III.2. Properties of the models with and without SB

To start let us analyse the original return series over the period 1995-2002.

6. Table: Descriptives of BUX return series

<table>
<thead>
<tr>
<th>Observations</th>
<th>1992</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.0838</td>
</tr>
<tr>
<td>Std.Devn.</td>
<td>1.9329</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.9655</td>
</tr>
<tr>
<td>Excess Kurtosis</td>
<td>12.601</td>
</tr>
<tr>
<td>Minimum</td>
<td>-18.033</td>
</tr>
<tr>
<td>Maximum</td>
<td>13.616</td>
</tr>
<tr>
<td>Normality test:</td>
<td>Chi^2(2) = 1858.3 [0.0000]**</td>
</tr>
<tr>
<td>Jarque-Bera test</td>
<td>t=13488 [0.00000]**</td>
</tr>
<tr>
<td>ARCH 1-2 test:</td>
<td>F(2, 1985) = 99.459 [0.0000]**</td>
</tr>
<tr>
<td>ARCH 1-5 test:</td>
<td>F(5, 1979) = 48.386 [0.0000]**</td>
</tr>
<tr>
<td>ARCH 1-10 test:</td>
<td>F(10, 1969) = 28.031 [0.0000]**</td>
</tr>
</tbody>
</table>

It is highly non-normal with negative skewness and very high leptokurtosis. There is also strong evidence of ARCH effect.

Now let us turn to the question whether we can “improve” those characteristics by estimating conditional models. In this chapter, we compare 5 models in terms of their parameter estimates, diagnostics and persistence implied by their GARCH parameters.

We compare the basic model (“basic” is an AR(1)-GARCH(1,1)) with the four SB models.

Comparing the basic and SB models, we find that the persistence captured by the sum of the ARCH and GARCH parameter decreases when SBs are accounted for. That finding is in line with the literature suggesting that persistence is overstated due to unaccounted SB. The fall is largest in SB4 (SP25 model).
We also compare the **unconditional mean and variance** implied by different models for each sub-period.\(^{30}\) Although in some cases the sub-periods are so small (25) that it makes the parameter estimates unreliable, even if that is kept in mind, the differences are striking.

The basic model masks, for example, the fact that the average daily return declined, close to zero, in the second half of the sample. It also distorts dramatically the level of unconditional variance, which was very high in some short sub-periods but much lower—than suggested by the basic model—in the majority of the sample.

When the **diagnostics** of the 5 models are compared, SB models seem to over-perform the basic model. The improvement is even more dramatic when compared to the original series. The standardised residuals of the SB models all have virtually zero mean and 1 standard deviation. Their skewness is zero, however they stay leptokurtic with excess kurtosis between 2.3 – 3. Normality is rejected in all 4 cases. They do not have any ARCH effect, or serial correlation. The Nyblom statistics does not suggest instability in any of the parameter estimates.

---

\(^{30}\) It is rather for illustrative purposes; therefore the exact periods are not shown. One can go back and find them in the definition of each SB model.
In contrast, the basic model’s standardised residual is still negatively skewed (-0.376), has higher excess kurtosis relative to the SB models (4.87). There is still some autocorrelation in standardised residuals at higher lags (Q(10) is significant). Although the ARCH F-test does not find any ARCH effect, the Q statistics on squared standardised residuals do reject no autocorrelation at 10%. According to the Nyblom statistics, the stability of the constant in both the mean and the GARCH equation can be questioned at 5%.

**III.3. In-sample forecasting performance**

First, in-sample forecast performance is evaluated using standard **statistical loss functions**, and then VAR back-tests are conducted.

9. **Table: In-sample forecasting performance for volatility**

<table>
<thead>
<tr>
<th></th>
<th>MAE</th>
<th>MSE</th>
<th>MAPE</th>
<th>AMAPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>SB1</td>
<td>181.8</td>
<td>4.25</td>
<td>352.197</td>
<td>57.9</td>
</tr>
<tr>
<td>SB2</td>
<td>183.8</td>
<td>4.33</td>
<td>370.216</td>
<td>57.5</td>
</tr>
<tr>
<td>SB3</td>
<td>184.7</td>
<td>4.24</td>
<td>320.598</td>
<td>57.2</td>
</tr>
<tr>
<td>SB4</td>
<td>168.8</td>
<td>4.13</td>
<td>286.705</td>
<td>57.2</td>
</tr>
<tr>
<td>Basic</td>
<td>195.0</td>
<td>4.46</td>
<td>378.462</td>
<td>58.2</td>
</tr>
<tr>
<td>EWMA</td>
<td>187.9</td>
<td>4.28</td>
<td>519.348</td>
<td>57.6</td>
</tr>
<tr>
<td>MA250</td>
<td>226.7</td>
<td>5.00</td>
<td>705.579</td>
<td>59.9</td>
</tr>
<tr>
<td>MA500</td>
<td>264.9</td>
<td>5.51</td>
<td>740.065</td>
<td>60.9</td>
</tr>
</tbody>
</table>

Based on the comparison of fitted and actual value\(^{31}\) of volatility, the SB models seem to outperform all the other models—except EWMA—by each measure.

Different statistical loss functions yield very similar ranking of models. Among the SB models, SB4 is the winner by each measure. SB models are followed by EWMA and the basic model. The worst performance is shown by moving average models.

---

\(^{31}\) The volatility is not observable, here the “actual” or ex post value of variance is defined as follows: \((r(t)-\mu)^2\), where \(\mu\) is the sample mean of return.
10. Table: Ranking by statistical loss functions

<table>
<thead>
<tr>
<th></th>
<th>MAE</th>
<th>MSE</th>
<th>MAPE</th>
<th>AMAPE</th>
<th>Average</th>
<th>Final</th>
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</thead>
<tbody>
<tr>
<td>SB1</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>5</td>
<td>3.3</td>
<td>3</td>
</tr>
<tr>
<td>SB2</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>3.5</td>
<td>4</td>
</tr>
<tr>
<td>SB3</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2.5</td>
<td>2</td>
</tr>
<tr>
<td>SB4</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1.0</td>
<td>1</td>
</tr>
<tr>
<td>Basic</td>
<td>6</td>
<td>6</td>
<td>5</td>
<td>6</td>
<td>5.8</td>
<td>6</td>
</tr>
<tr>
<td>EWMA</td>
<td>5</td>
<td>5</td>
<td>6</td>
<td>4</td>
<td>5.0</td>
<td>5</td>
</tr>
<tr>
<td>MA250</td>
<td>7</td>
<td>7</td>
<td>7</td>
<td>7</td>
<td>7.0</td>
<td>7</td>
</tr>
<tr>
<td>MA500</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8.0</td>
<td>8</td>
</tr>
</tbody>
</table>

Regarding their VAR forecasts, the performance of various models depends on p. When the number of hits (cases when loss exceeds VAR) are counted, at p=5% only one model leads to higher then the theoretical rate of hits. However, for 1% and 0.5% none of the models ensures VAR high enough to cover losses at those probabilities.

11. Table: VAR forecasting performance (failures highlighted)

<table>
<thead>
<tr>
<th>P</th>
<th>model</th>
<th>exceed%</th>
<th>Kupiec</th>
<th>Christoffersen</th>
</tr>
</thead>
<tbody>
<tr>
<td>p=5%</td>
<td>Basic</td>
<td>4.52</td>
<td>1.01</td>
<td>6.03</td>
</tr>
<tr>
<td></td>
<td>SB1</td>
<td>5.07</td>
<td>0.02</td>
<td>5.84</td>
</tr>
<tr>
<td></td>
<td>SB2</td>
<td>4.77</td>
<td>0.23</td>
<td>4.10</td>
</tr>
<tr>
<td></td>
<td>SB3</td>
<td>4.42</td>
<td>1.48</td>
<td>9.08</td>
</tr>
<tr>
<td></td>
<td>SB4</td>
<td>4.42</td>
<td>1.48</td>
<td>2.57</td>
</tr>
<tr>
<td></td>
<td>EWMA</td>
<td>4.47</td>
<td>1.23</td>
<td>16.01</td>
</tr>
<tr>
<td></td>
<td>MA250</td>
<td>4.25</td>
<td>2.18</td>
<td>15.87</td>
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<tr>
<td></td>
<td>MA500</td>
<td>4.29</td>
<td>1.66</td>
<td>11.88</td>
</tr>
<tr>
<td>p=1%</td>
<td>Basic</td>
<td>1.51</td>
<td>4.46</td>
<td>11.18</td>
</tr>
<tr>
<td></td>
<td>SB1</td>
<td>1.61</td>
<td>6.25</td>
<td>15.53</td>
</tr>
<tr>
<td></td>
<td>SB2</td>
<td>1.46</td>
<td>3.66</td>
<td>4.26</td>
</tr>
<tr>
<td></td>
<td>SB3</td>
<td>1.46</td>
<td>3.66</td>
<td>10.76</td>
</tr>
<tr>
<td></td>
<td>SB4</td>
<td>1.61</td>
<td>6.25</td>
<td>6.63</td>
</tr>
<tr>
<td></td>
<td>EWMA</td>
<td>1.81</td>
<td>10.58</td>
<td>23.40</td>
</tr>
<tr>
<td></td>
<td>MA250</td>
<td>1.84</td>
<td>9.88</td>
<td>15.22</td>
</tr>
<tr>
<td></td>
<td>MA500</td>
<td>2.08</td>
<td>13.35</td>
<td>22.03</td>
</tr>
<tr>
<td>p=0.5%</td>
<td>Basic</td>
<td>1.26</td>
<td>16.05</td>
<td>34.26</td>
</tr>
<tr>
<td></td>
<td>SB1</td>
<td>1.26</td>
<td>16.05</td>
<td>25.43</td>
</tr>
<tr>
<td></td>
<td>SB2</td>
<td>1.16</td>
<td>12.50</td>
<td>13.74</td>
</tr>
<tr>
<td></td>
<td>SB3</td>
<td>1.31</td>
<td>17.95</td>
<td>26.28</td>
</tr>
<tr>
<td></td>
<td>SB4</td>
<td>1.10</td>
<td>10.86</td>
<td>12.23</td>
</tr>
<tr>
<td></td>
<td>EWMA</td>
<td>1.41</td>
<td>21.97</td>
<td>34.30</td>
</tr>
<tr>
<td></td>
<td>MA250</td>
<td>1.32</td>
<td>16.21</td>
<td>20.61</td>
</tr>
<tr>
<td></td>
<td>MA500</td>
<td>1.47</td>
<td>16.51</td>
<td>21.04</td>
</tr>
</tbody>
</table>
According to Kupiec’s proportion of failures test\textsuperscript{32} none is rejected at 5%, but all the models fail at 0.5%. At 1% two SB and the basic models are still acceptable. However, according to the conditional test (Christoffersen) all the MA models fail at all p; moreover, the basic model passes the test only at 5%. SB2 and SB4 with h=25 is accepted even at 1%.

When the ranking based on the value of the Christoffersen test\textsuperscript{33} across different confidence levels is investigated, the following picture emerges: surprisingly EWMA is the worst performer at almost all confidence level; it is followed by the two MA models; the ranking of the basic model sharply deteriorates with p; overall the SB models perform well, but their performance varies across confidence levels. The best performers across all p are the SB models with h=25 (SB2 and SB4).

7. Graph: Ranking according to the Christoffersen test

The above results based on in-sample forecasting performance are very promising. However, best fit in-sample does not imply good out-of-sample forecasting performance. The next chapter extends the analysis in this direction.

\textbf{IV. Results of rolling-window estimation}

To do out-of-sample forecast we mimic the actual practice of banks and the requirements of Basel regulation. We estimate SBs and the various conditional and un-

\textsuperscript{32} For rejection we use 97.5% and 2.5% critical values. As the chi-square test is two-sided this corresponds to p=5%.

\textsuperscript{33} Rank 1 is attached to all the models which are not rejected. The rank of the others depends on the value of the test statistic. Should some fall below the lower critical value, those would be preferred to the ones which exceed the upper critical value—those which result in too high VAR are preferred to those resulting in too low VAR. Actually none of the models failed due to low value of the test statistic.
conditional model parameters over 2y’s (500) observations. Then we calculate 125 out-of-sample static volatility- and VAR forecasts. The estimation window is then rolled forward by 125 observations and the whole process is repeated. Altogether we have 12 estimation periods (P1 - P12) and 12*125=1500 forecasts.

All the significant breaks, found by either IC or SP with h=25, 50, 100, are considered. Different models are compared by sub-periods, but I also combine them to obtain forecasts for the “entire” forecast period (T-500). As a result, the following SB models are constructed:

### 12. Table: Combined SB models to cover the entire period

<table>
<thead>
<tr>
<th>Model</th>
<th>BIC100</th>
<th>SP100</th>
<th>BIC50</th>
<th>SP50</th>
<th>BIC25</th>
<th>SP25</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1_M1</td>
<td>P1_M2</td>
<td>P1_M3</td>
<td>P1_M4</td>
<td>P1_M3</td>
<td>P1_M4</td>
<td></td>
</tr>
<tr>
<td>P2_M1</td>
<td>P2_M2</td>
<td>P2_M3</td>
<td>P2_M3</td>
<td>P2_M3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>P3_M1</td>
<td>P3_M2</td>
<td>P3_M3</td>
<td>P3_M4</td>
<td>P3_M5</td>
<td>P3_M6</td>
<td></td>
</tr>
<tr>
<td>P4_M1</td>
<td>P4_M2</td>
<td>P4_M3</td>
<td>P4_M2</td>
<td>P4_M4</td>
<td>P4_M4</td>
<td></td>
</tr>
<tr>
<td>P5_M1</td>
<td>P5_M2</td>
<td>P5_M3</td>
<td>P5_M4</td>
<td>P5_M5</td>
<td>P5_M6</td>
<td></td>
</tr>
<tr>
<td>BASIC6</td>
<td>BASIC6</td>
<td>P6_M1</td>
<td>P6_M3</td>
<td>P6_M2</td>
<td>P6_M4</td>
<td></td>
</tr>
<tr>
<td>BASIC7</td>
<td>P7_M1</td>
<td>BASIC7</td>
<td>P7_M1</td>
<td>P7_M2</td>
<td>BASIC7</td>
<td></td>
</tr>
<tr>
<td>P8_M1</td>
<td>P8_M1</td>
<td>P8_M2</td>
<td>P8_M3</td>
<td>P8_M4</td>
<td>P8_M5</td>
<td></td>
</tr>
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<td>BASIC9</td>
<td>BASIC9</td>
<td>BASIC9</td>
<td>BASIC9</td>
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<td>BASIC10</td>
<td>BASIC10</td>
<td>BASIC10</td>
<td>P10_M1</td>
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</tr>
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<td>BASIC11</td>
<td>P11_M1</td>
<td>BASIC11</td>
<td>P11_M1</td>
<td>P11_M2</td>
<td>P11_M1</td>
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</tr>
<tr>
<td>P12_M1</td>
<td>P12_M2</td>
<td>P12_M3</td>
<td>P12_M3</td>
<td>P12_M4</td>
<td>P12_M5</td>
<td></td>
</tr>
</tbody>
</table>

Where no break was found by the given method (for example, BIC with h=100 has not found any break in period 7), the estimation and forecasts of the basic AR-GARCH model are used (BASIC7). In some cases, different SB models detected exactly the same breaks (as in period 12, where BIC50 and SP50 yield the same model: M3).

The forecasting performance is compared by the same measures as before, however here more emphasis is placed on the time dimension of relative performance.

### IV.1. Structural breaks found

The rolling-window exercise yields very similar results to the estimation based on the entire period. Obviously, as the window moves along the sample the SBs enter and leave with some lag (the SB close to the end of the sample is not detected yet).
The only “odd-one” is the break in mean found at the end of 1995, which is not detected when tests are run on the entire period. That corresponds to the start of the strong upward trend in the index, lasting until the Asian crisis.

9. Graph: SB in volatility found by the two estimation methods (rolling window vs entire period)

The breaks in volatility detected by the two methods are very similar as well.

IV.2. The basic versus the SB models

When the basic and SB models are compared, the findings are in line with the expectations. SB models usually have much smaller persistence than the basic models
—except in some turbulent periods, where the volatility process is very close to being integrated. The next graph highlights those differences.

10. Graph: Persistence in the basic and SB models

The **standardised residuals** of the SB models show superior properties relative to the basic model. Although \( N(0,1) \) is assumed to apply for standardised residuals, normality is rejected by the Jarque-Berra test for all models for all sub-periods—except for some SB models in P12. Nevertheless, there is an improvement over time: for the basic model, after reaching its highest value in 1997-1998, both the skewness and kurtosis\(^{35}\) decline. Moreover, SB models usually have thinner tails and are less asymmetric than the basic model. In this sense, fitting SB models make the standardised residual get closer to normality.\(^{36}\)

\(^{34}\)It is interesting to have a look at the changes in the parameters of the basic GARCH model. Typically the \( \alpha_1 \) and \( \beta_1 \) falls between 0-20 and 80-100 respectively. From 1999, that is true for the BUX return, however, in the preceding periods those parameter estimates are rather odd.

\(^{35}\)When the data is normally distributed both the value of skewness and kurtosis equal zero. Kurtosis for \( N(0,1) \) equals 3, but most of the software adjust it by deducting 3 from the calculated statistics.

\(^{36}\)Recall that we assume normality of the standardised residuals when VAR is calculated.
IV.3. Out-of-sample forecasting performance

Over the entire forecast horizon almost all the models fail according to the Christoffersen test (except BIC50 at p=5%), but at 5% none is rejected by the Kupiec test. As the Christoffersen test proved to be more conservative and test for independence as well, hereafter only that one is used.

13. Table: Models rejected (1) by POF tests

<table>
<thead>
<tr>
<th></th>
<th>ma250</th>
<th>ma500</th>
<th>ewma</th>
<th>Basic</th>
<th>BIC100</th>
<th>SP100</th>
<th>BIC50</th>
<th>SP50</th>
<th>BIC25</th>
<th>SP25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kupiec5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Kupiec1</td>
<td>✗</td>
<td>✗</td>
<td>✗</td>
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<td>✗</td>
<td>✗</td>
<td>✗</td>
<td>✗</td>
</tr>
<tr>
<td>Kupiec0.5</td>
<td>✗</td>
<td>✗</td>
<td>✗</td>
<td>✗</td>
<td>✗</td>
<td>✗</td>
<td>✗</td>
<td>✗</td>
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<td>Christ5</td>
<td>✗</td>
<td>✗</td>
<td>✗</td>
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<td>✗</td>
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</tr>
<tr>
<td>Christ1</td>
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<td>✗</td>
<td>✗</td>
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<td>✗</td>
</tr>
<tr>
<td>Christ0.5</td>
<td>✗</td>
<td>✗</td>
<td>✗</td>
<td>✗</td>
<td>✗</td>
<td>✗</td>
<td>✗</td>
<td>✗</td>
<td>✗</td>
<td>✗</td>
</tr>
</tbody>
</table>

Nevertheless, performance of the models varies through time and across p. When forecasts for 1 year (250) are considered, many models pass the test in certain periods. At p=5% rejections are more dispersed. Models are rejected in several periods (1997, 1998, 1999, 2001). At p=1% rejections occur almost exclusively in 1997 and 1998 (only 2 SB and the EWMA models are not rejected there), the same applies to p=0.5%, but here some MA are rejected in 2001 as well.

Overall, SB models slightly perform better than the alternatives, but the difference is not material. Why is this so? One of the reasons may be that we expect SB models to provide
better forecasts only in periods when shifts occurred in the estimation period but not in the forecast period. That applies only for 2000 and 2002. However, in both years all the models passed the tests, thus one cannot rank them.

14. Table: Model rejection based on backtest on 1 year (250) forecasts

<table>
<thead>
<tr>
<th></th>
<th>Ma250</th>
<th>ma500</th>
<th>ewma</th>
<th>basic</th>
<th>BIC100</th>
<th>SP100</th>
<th>BIC50</th>
<th>SP50</th>
<th>BIC25</th>
<th>SP25</th>
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<tbody>
<tr>
<td>1997</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>0</td>
<td>0</td>
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<td>1998</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
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<td>0</td>
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<tr>
<td>1999</td>
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<td>0</td>
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<td>2000</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

We also looked at the rank among models based on the Statistical loss functions, the Expected Shortfall and the Christoffersen test. Again, based on the entire forecast there is some rather weak evidence on the superior performance of SB models. Models’ rank varies across evaluation criterions and p. Although no obvious ranking emerges, some findings might be drawn based on the overall performance:

- The average value of losses above the capital (ES) is much lower for SB models, but only at higher probability. The ranking at p=5% is very different from the ranking at lower p.
- The ranking by the backtest versus EES are often very different.
- Across various evaluation criterion MA500 seems to be the worst, however interestingly MA250 does not perform so badly.
- BASIC is the worst model according to the backtest.

37 In 2000, there was one SB detected but very close to the end of the year.
SB models usually get more good ranks and less bad ranks relative to the other models. Among them, BIC25 show good performance according to the loss functions, however results in high ES relative to the other models.

15. **Table: Ranking of models**

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Statistical loss function</th>
<th>Christoffersen</th>
<th>“Expected Excess Shortfall”</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>5%</td>
<td>1%</td>
</tr>
<tr>
<td>Basic</td>
<td>5</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>EWMA</td>
<td>2</td>
<td>9</td>
<td>5</td>
</tr>
<tr>
<td>MA250</td>
<td>5</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>MA500</td>
<td>9</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>SP100</td>
<td>3</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>BIC100</td>
<td>10</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>SP50</td>
<td>4</td>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>BIC50</td>
<td>8</td>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td>SP25</td>
<td>7</td>
<td>8</td>
<td>1</td>
</tr>
<tr>
<td>BIC25</td>
<td>1</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

The first column is based on the average rank of the 4 statistical loss functions. Nonetheless, the average masks large differences in ranking by various statistical loss functions. The only exception is again BIC25, which shows superior performance by each measure. One might argue that the differences between MAE and MSE suggest that MA and basic perform better most of the time, but with a few major errors (the impact of large forecast errors are amplified by squaring in MSE, but not in MAE where the absolute value is used). However, we could not find any reasonable explanation for the other differences.

16. **Table: Ranking according to various statistical loss functions**

<table>
<thead>
<tr>
<th></th>
<th>MAE</th>
<th>MSE</th>
<th>MAPE</th>
<th>AMAPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>basic</td>
<td>5</td>
<td>6</td>
<td>10</td>
<td>3</td>
</tr>
<tr>
<td>ewma</td>
<td>2</td>
<td>8</td>
<td>2</td>
<td>1</td>
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<tr>
<td>ma250</td>
<td>3</td>
<td>9</td>
<td>3</td>
<td>9</td>
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<tr>
<td>ma500</td>
<td>4</td>
<td>10</td>
<td>6</td>
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<tr>
<td>SP100</td>
<td>7</td>
<td>5</td>
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<tr>
<td>BIC100</td>
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</tr>
<tr>
<td>SP50</td>
<td>6</td>
<td>2</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>BIC50</td>
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<td>4</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>SP25</td>
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<td>3</td>
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<td>5</td>
</tr>
<tr>
<td>BIC25</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

To illustrate the variability of performance through time, we plotted the moving sum of exceedences and EES over 250 days for the different models. Each reach its peak in
1998-1999, due to the big jumps found in 1997-1998, which cause the performance of the unconditional models, but to a lesser extent even the conditional models to decline. During the last two years of the sample not much difference between various models can be seen, mainly because the magnitude and frequency of SB is much lower after 1998. To start with the number of hits over a year, we use the case with p=5% to illustrate the differences. However, the series show very much the same pattern for other p’s. First, the MA models (in particular MA500) show very poor performance: the number of hits fluctuates very widely, reaching its maximum (27) in 1998, but even drops to zero in 2000. As we know, this method is very sensitive to extreme observations, and moreover, there are abrupt changes not only when those extremes move in but also when they move out from the estimation sample. It is also striking that the basic model has the largest hits in the critical 1998-1999 period. This is due to the presence of SB. Overall, SB models and the EWMA outperform the others in the sense that the number of hits stays in a rather narrow band around the theoretical value. The following graphs show the results in different groups.

12. **Graph: Moving sum of hits over 250 days at p=5%**

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38 As we use 250 observations, at p=5% the theoretical value of hits is 12.5. On the graphs that is represented by the dashed line.
As to the average value of hits over VAR (EES), there is not much difference among models, when the sum of excess shortfall is divided by the number of hits. That means given the shortfall occur, its expected value (over VAR) through time does not differ significantly among models. They all are at a high level in 1998-1999, then drop to a much lower level.
However, when the sum of shortfalls over 250 days is considered, the differences are more striking. In the critical 1998-1999 period SB models produce much lower losses over VAR. Those differences are nevertheless due to the differences in the frequency of hits. That is, MA and basic models relative worth performance is due to the high frequency of exceedences rather than the average value of excess shortfall.
17. Graph: Moving sum of EES over 250 days at p=5%

18. Graph: Moving sum of EES over 250 days at p=5%; SB models
V. Conclusion

Using the Bai-Perron algorithm we detected several breaks in the mean and volatility of the BUX return over the period 1995-2002. There was a significant decrease in unconditional mean around 1997-1998. Evolving market efficiency and the halt in a strong upward trend are likely to explain that shift. Regarding volatility, several upward shifts in its unconditional level (1996, 1997, 1998) were located. Increased trading volume, the impact of the Russian and Asian crises and even bad luck might explain those changes. Some other SBs found signal rather regime changes than longer run level shift (Autumn 1997, 1998). The changes in unconditional mean and volatility do support the presence of shifts, thus underlining the need for modelling SB.

When SBs are explicitly taken into account in our statistical models, the persistence decline sharply, implying faster adjustment to new information, than suggested by the basic AR-GARCH model. The diagnostics of the standardised residuals improve, growing closer to normality, relative to the basic AR-GARCH model, even more to the original return series.

In-sample the models with SB outperform all the MA and basic models (but not EWMA) according to all the statistical and VAR loss functions. Although at higher probability all the models fail, at lower probability SB models perform better. Their relative performance (ranks based on test statistics) seem to be superior as well. However, out-of-sample forecasting yields more mixed results. The relative performance of various models depends on the loss function used and the chosen probability level (p). The large number of breaks detected makes it difficult to highlight the differences in relative performance as there are very few cases where there is no SB in the forecasting period. Nevertheless, we do find some evidence on the superior performance of SB models. They lead to lower average loss over VAR, result in less rejection by the backtest, and overall they are assigned more good ranks than the other models. Based on the graphical analysis on the moving sum of hits (failures of models) SB and EWMA models should be favoured, as they fluctuate in a much narrower band. Their superiority is more pronounced in the turbulent periods of 1997-1998.

These results shed some light on the importance of detecting SBs and incorporating them into our statistical models. In general, they add to our knowledge about the behaviour of the stock market and about its changes. Inclusion of SB also has advantages when used in financial risk management. However, the mixed results of forecast evaluation warn of the difficulties involved in using various tests as a means of model selection.
References

• Basel Committee on Banking Supervision (1996) “Amendment to the Capital Accord to incorporate market risks”
• Basel Committee on Banking Supervision (2001) “Principles for the management and supervision of interest rate risk” Consultative Document
• J. Berkowitz and J. O’Brien (200?) “How Accurate are Value-at-Risk Models at Commercial Banks?”
volatility and stock market development: Evidence from Spain” Journal of Banking Finance 28

M.Guidolin and A. Timmermann (2003) “Value at Risk and Expected Shortfall under Regime Switching”
P.Hartmann, S.Straetmans and C.G.De Vries (2001) “Asset market linkages in crisis periods” ECB WP 71
Spring 2002

- J.A. Lopez “Regulatory Evaluation of Value-at-Risk Models” FED NY
- C. Morana, and A. Beltratti (2002) “Structural Change and Long Range Dependence in Volatility of Exchange Rates: Either, Neither or Both?” mimeo
• M.H. Pesaran and A. Timmermann (2003) “How costly is to ignore breaks when forecasting the direction of a time series” International Journal of Forecasting
Why?” mimeo

- T.Vorst ( ) “Optimal Portfolios under a Value at Risk Constraint”
- J.Wang and E.Zivot “A Time Series Model of Multiple Structural Changes in Level, Trend and Variance”
1995/1
SIMON, András: Aggregált kereslet és kínálat, termelés és külkereskedelem a magyar gazdaságban 1990-1994
Aggregate Demand and Supply, Production and Foreign Trade in the Hungarian Economy, 1990-1994 (available only in Hungarian)

1995/2
NEMÉNYI, Judit: A Magyar Nemzeti Bank devizazadósságán felhalmozódó árfolyamveszteség kérdései
Issues of Foreign Exchange Losses of the National Bank of Hungary (available only in Hungarian)

1995/3
KUN, János: Seignorage és az államadóság terhei
Seignorage and the Burdens of Government Debt (available only in Hungarian)

1996/1
SIMON, András: Az infláció tényezői 1990-1995-ben
Factors of Inflation, 1990-1995 (available only in Hungarian)

1996/2
NEMÉNYI, Judit: A tőkebeáramlás, a makrogazdasági egyensúly és az eladósodási folyamat összefüggései a Magyar Nemzeti Bank eredményének alakulásával
The Influence of Capital Flow, Macroeconomic Balance and Indebtedness on the Profits of the National Bank of Hungary (available only in Hungarian)

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SIMON, András: Sterilizáció, kamatpolitika az államháztartás és a fizetési mérleg
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The Structure, Level and Development of Consumer Prices in Hungary, 1991-1996 — An Econometric Analysis Based on the Detailed Consumer Price Index (available only in Hungarian)

1996/6
Bank Financing of Enterprises in Hungary, 1991-1994 (available only in Hungarian)

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CSERMELY, Ágnes: Az inflációs célkitűzés rendszere
The Inflation Targeting Framework (available only in Hungarian)

1997/2
VINCZE, János: A stabilizáció hatása az árakra, és az árak és a termelés (értékesítés) közötti összefüggésekre
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Capital Inflow, Sterilization and the Quantity of Money

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Savings and Portfolio Decisions of Hungarian Households, 1980-1996 (available only in Hungarian)

1997/5
ÁRVAI, Zsófia: A sterilizáció és tőkebeáramlás ökonometriai elemzése
An Econometric Analysis of Capital Inflows and Sterilization (available only in Hungarian)

1997/6
ZSOLDOS, István: A lakosság Divíziás-pénz tartási viselkedése Magyarországon
Characteristics of Household Divisia Money in Hungary (available only in Hungarian)

1998/1
ÁRVAI, Zsófia — VINCZE, János: Valuták sebezhetősége: Pénzügyi válságok a ’90-es években
Vulnerability of Foreign Currency: Financial Crises in the 1990s (available only in Hungarian)

1998/2
CSÁJBÓK, Attila: Zéro-kupon hozamgörbe becslés jegybanki személyből
Zero-coupon Yield Curve Estimation from a Central Bank Perspective

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