Imperfect Competition in Financial Markets and Capital Structure

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Abstract

We consider a model of corporate finance with imperfectly competitive financial intermediaries. Firms can finance projects either via debt or via equity. Because of asymmetric information about firms’ growth opportunities, equity financing involves a dilution cost. Nevertheless, equity emerges in equilibrium whenever financial intermediaries have sufficient market power. In the latter case, best firms issue debt while the less profitable firms are equity-financed. We also show that strategic interaction between oligopolistic intermediaries results in multiple equilibria. If one intermediary chooses to buy more debt, the price of debt decreases, so the best equity-issuing firms switch from equity to debt financing. This in turn decreases average quality of equity-financed pool, so other intermediaries also shift towards more debt.

Keywords: capital structure, pecking order theory of finance, oligopoly in financial markets, second degree price discrimination

JEL Codes: D43, G32, L13

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1 Introduction

The choice of capital structure is one of the central issues in corporate finance. The cornerstone paper by Modigliani and Miller (1958) established that capital structure is irrelevant so long as financial markets are perfect. As financing decisions do matter in the real world, corporate finance literature has advanced a number of theories that show how various imperfections explain the observed patterns of capital structure. These explanations have mostly concentrated on the imperfections on the side of the firm: the optimal capital structure minimizes the costs borne by investors as a result of taxes, asymmetric information, conflicts of interest between management and shareholders, etc. Since the financial markets are assumed to be perfectly competitive, these costs are passed back to the firm in the form of a higher cost of capital, thus providing incentives to choose an optimal capital structure.

In this paper, we study how the capital structure is affected by an imperfection on the side of financial markets. We assume that financial intermediaries have market power. There are many reasons to believe that financial markets are not perfectly competitive. Financial services require reputational capital; information accumulation and processing also create economies of scale and barriers to entry (Dell Arricia et al. 1999). Morrison and Wilhelm (2007) argue that the increasing codification of certain investment banking activities have recently resulted in even greater scale economies in the investment banking business.

Not surprisingly, after the Glass-Steagall Act was repealed in 1999, the global financial market has been increasingly dominated by a few “global, universal banks of new generation” (Calomiris 2002) that provide both commercial and investment banking services (as well as other financial services). These banks also command a substantial market share in virtually all financial markets, including debt and equity issues. In 2007, according to Thomson Reuters, the nine largest financial groups (Goldman Sachs, Lehman Brothers, Merrill Lynch, Morgan Stanley, Citi, JP Morgan, Credit Suisse, Deutsche Bank, and UBS) controlled more than 50% in every major financial market; in many markets the top five financial intermediaries controlled up to 70% of the market. It is too early to judge the effect of the 2008 crisis on the financial market structure but it has certainly increased the remaining top banks’ market shares. Morrison and Wilhelm (2007) cite Securities Data Corporation’s data to show that the top five (top ten) banks’ share in the US common stock offering rose from 38% (62%) in 1970 to 64% (87%) in 2003. These trends have not been unnoticed by policymakers and academics. In 1999, the US Department of Justice launched an antitrust investigation on the IPO fees (Smith 1999). The
academic debate on the collusive nature of the clustering of the IPO fees is not conclusive (see Chen and Ritter 2000 who argue that the fees’ clustering around 7% in the US is a sign of tacit collusion and Torstila 2003 for cross-country evidence and the summary of the debate). Yet, the very nature of this debate suggests that the investment banking industry is not perfectly competitive. This conjecture is also consistent with the legal analysis by Griffith (2005) who argues that underwriters possess market power and use it for price discrimination.

Why does imperfect competition matter for capital structure? Once the financial intermediaries start to behave strategically, the logic of conventional capital structure theories falls apart. Under perfect competition, the investors’ costs are passed onto the firm because investors earn zero rents on all financial instruments. In this paper, we still assume that investors are perfectly competitive, but the intermediaries between investors and firms are oligopolistic. Therefore, financial intermediaries receive positive rents; these rents may differ for debt and equity investments. Since firms choose capital structure depending on their privately known growth opportunities, intermediaries can use capital structure as a means of the second degree price discrimination (similarly to using monetary and barter contracts in Guriev and Kvassov 2004). The purpose of discrimination is to extract higher fees from more profitable firms. We find that equilibrium capital structure is different in competitive and concentrated markets. For expository clarity, we assume away all possible costs of debt financing. In this case, in line with the pecking order theory, debt crowds out equity as long as financial markets are sufficiently competitive. However, as markets become more concentrated, equity financing does emerge in equilibrium. Concentration of market power results in a substantial wedge between the oligopolistic interest rate and intermediaries’ cost of funds. Hence, there is a pool of firms that would borrow at rates which are below the market interest rate on debt but still above intermediaries’ cost of funds. In order to serve these firms without sacrificing revenues from lending at a high rate to existing borrowers, intermediaries use capital structure as a screening device. The better firms still prefer debt, while the less profitable firms are happy to issue equity. Therefore, the model is consistent with the observed increase in concentration in investment banking and the rise of equity issues worldwide in recent years.

What makes our paper more than just another model of capital structure is the study of strategic interaction that results in multiple equilibria. As we show, these equilibria differ in terms of both capital structure and asset prices even though all agents are fully rational. This in turn provides a very simple rationale for stock market volatility, bubbles and crashes without resorting to assumptions on bounded rationality or limits of arbitrage.
The intuition for multiplicity of equilibria is the strategic complementarity of portfolio choices by the financial intermediaries.\(^1\) Suppose that one intermediary decides to move from debt to equity. This raises the interest rate on debt so that some firms that used to borrow can no longer afford debt finance. These firms switch to equity which improves average quality of the pool of equity-financed firms (all debt-financed firms are better than equity-financed ones). This makes equity investment more attractive so other investors also choose to shift from debt to equity. We show that multiple equilibria do exist for a range of parameter values.

Our analysis has two main empirical implications. First, *ceteris paribus* both across countries and over time, a higher concentration of financial market power should result in a greater reliance on equity finance. Second, there may emerge multiple stable equilibria; in each equilibrium stock prices are based on fundamentals, and investors buy debt and equity based on their rational beliefs. Hence, either equilibrium is not a temporary bubble but is sustainable in the long run. Our theory predicts that multiple equilibria emerge only in the intermediate ranges of concentration of the financial market power. If markets are perfectly competitive, there is a unique equilibrium where debt finance prevails; if markets are very concentrated, there is only one equilibrium with a high share of equity financing.

Both predictions, however, are hard to test as there are many other determinants of capital structure that are correlated with changes in concentration of the financial markets. In particular, the cross-country test of our hypothesis is problematic as legal protection of outside shareholders in the US results in a widespread use of equity even though the US financial markets are very competitive (La Porta et al. 1998). As for the within-US experience over time, it is rather consistent with our results: the consolidation of financial industry in 1990s was accompanied by a growth in equity finance and in higher stock market volatility. In any case, finding appropriate instruments or locating a suitable natural experiment is a subject for future empirical work.

*Related literature.* Market concentration is not the only explanation for the coexistence of debt and equity under asymmetric information. In the pecking order literature equity finance may emerge in equilibrium either if debt is costly or if information production is endogenous. In Bolton and Freixas (2000), both bank loans and public debt coexist in equilibrium with equity. Although equity financing involves a dilution cost, it still emerges in equilibrium since debt financing is also costly. Banks need to raise funds themselves and, therefore, bear intermediation costs, while bond financing involves inefficient liquidation.

\(^1\)Our model is an application of the Bulow et al. (1985) multi-market oligopoly model in the case where demands rather than costs are interrelated across markets.
Since dispersed bondholders cannot overcome the free-rider problem, they are less likely to be flexible ex post (unlike banks). Again, each firm chooses the capital structure which is the least costly one for the investors since the perfect competition in financial markets translates investors’ costs into a higher cost of capital for the firm. Other potential costs of excessive leverage include costs of bankruptcy and agency costs of debt (Bradley et al. 1984). Cooney and Kalay (1993) consider the case of asymmetric information about both mean and volatility of the project returns; equity finance emerges in equilibrium. In Boot and Thakor (1993) and in Fulghieri and Lukin (2001), equity issues provide incentives for investors to produce information, hence bringing stock price closer to fundamentals and increasing issuer’s revenues.

Our model is based on the pecking order theory of capital structure (Myers and Majluf 1984). There is no consensus in the literature whether the pecking order theory outperforms the other explanations of capital structure, the trade-off theory and the agency theory. The empirical literature produces controversial results (see e.g. Myers 2001, Baker and Wurgler 2002, Mayer and Sussman 2004, Welch 2004, Fama and French 2005). It would probably be safe to say (see e.g. Fama and French 2002, and Leary and Roberts 2007) that a simple pecking order theory is certainly outperformed by the “complex pecking order theory” which incorporates features of the other theories. While we use the original pecking order theory as a point of reference, our results certainly extend to more general setups (see Sections 4 and 5). Moreover, our analysis shows that even the simple pecking order theory may be consistent with the data once the imperfect competition in financial markets is taken into account. Once the perfect competition assumption is relaxed, equity is issued even in this simple setup with all potential costs of debt financing assumed away.

While most of the capital structure literature studies perfectly competitive financial markets, there are a few papers that focus on imperfect competition. Petersen and Rajan (1994, 1995) consider a model of a monopolistic creditor that performs better than competitive market because it is able to form long-term ties and internalize the debtor’s benefits from investment. In many ways, this arrangement is similar to our equity financing (which also emerges in highly concentrated markets). Faulkender and Petersen (2006) also focus on the imperfections on the market’s side and show that underleverage may be related to rationing by lenders rather than to firms’ characteristics. Neither paper, however, considers oligopoly and therefore does not describe the effects of strategic interactions.

The paper by Degryse et al. (2009) also studies imperfect competition in banking and focuses on the interaction between organizational structure and the imperfectly competi-
tive equilibrium. Our setup is similar but we focus on the interaction between debt and equity markets while Degryse et al. only consider lending.

There is also a literature on market microstructure (e.g. Brunnermeier 2001, ch. 3) that explicitly models the competition between market makers in financial markets. Our setting is most similar to Biais et al. (2000) who consider oligopolistic uninformed market makers screening informed traders. However, the market microstructure models study a single financial market while we focus on the situation where financial intermediaries interact strategically in two markets (debt and equity) using capital structure to screen firms.

Our paper is also related to the literature on bubbles and crashes, as well as the one on the IPO waves. While our model does not describe bubbles (defined as deviations of stock prices from their fundamental values), we do show that there are multiple equilibria with different stock returns and volumes of stock issued; in this respect our paper is similar to Abreu and Brunnermeier (2003) who explain persistent bubbles by strategic interaction between rational arbitragers over time. Also, our model provides a rational explanation for the widespread market timing; the fact that firms issue equity when stock price is high and repurchase when low (Baker and Wurgler 2002) can be explained by multiplicity of equilibria.

The rest of the paper is structured as follows. In Section 2 we set up the model. In Section 3 we fully characterize the equilibria in a special case where the distribution of firms satisfies the monotone hazard rate condition; we show that if market structure is sufficiently concentrated there may be two stable equilibria: one with both debt and equity finance, and the other with debt finance only. We also consider an example without the monotone hazard rate condition where there are multiple equilibria with equity. Section 4 generalizes the model to the setting with agency costs and continuous choice of capital structure. Section 5 discusses further extensions. Section 6 describes empirical implications and concludes.

2 The model

2.1 The setting

There are two periods: \textit{ex ante} $t = 1$ (financing and investments) and \textit{ex post} $t = 2$ (realization of returns and payoffs). Discount rates are normalized to 1. There are a finite number of investors and a continuum of firms of measure 1.

\textit{Firms}. Each firm has an individual investment project that requires 1 unit of funds
in the first period and brings gross return $\pi$ in the second period. The return is the firm’s private information at the time of financing but is publicly observable ex post. The firms’ types $\pi$ are distributed on $[\pi, \bar{\pi}]$ with c.d.f. $F(\cdot)$ without mass points. Firms have no cash and, therefore, have to rely on either debt or equity.\footnote{We rule out a possibility of issuing both equity and debt until Section 4. One can assume that each method of finance may involve a fixed cost, say the same for debt and equity.}

Firms act in the interests of their existing shareholders. After production takes place and payoffs are realized, firms are liquidated. The firms’ outside options are normalized to zero.

\textit{Intermediaries}. There are $N$ financial intermediaries. The intermediaries have unlimited access to investors’ funds at a constant cost $\rho$ (e.g., an interest rate to be paid to the ultimate providers of funds). The intermediaries can choose how much to invest in bonds or stocks. The intermediaries have market power and behave strategically; they take into account the impact of their strategies on the market prices of debt and equity.

It is important to emphasize that continuum of firms and a finite number of intermediaries does not imply that intermediaries are scarce and projects are in infinite supply. On the contrary, the number of projects is limited (normalized to 1), and intermediaries can bring in an unlimited amount of resources (at a marginal cost $\rho$).

\textit{Debt}. In this model we do not distinguish between bank loans and bonds. The debt contract is standard: “borrow $D$ in the first period, pay back $rD$ in the second period; if the repayment is not made, the creditors take over the firm.” The return on debt, $r$, is endogenous and is determined in an (imperfectly competitive) market equilibrium model.

We assume that there is an infinitesimal cost of bankruptcy. If the firm is indifferent between repaying or undergoing bankruptcy, it always chooses repayment. In the first period, the firm has no uncertainty about its second-period returns. As a result, the firm never borrows more than it can pay back and default on debt never happens. The first-period price of a debt contract that promises to pay the investor $1$ is, therefore, $p = 1/r$.

\textit{Equity}. The equity market is a market for individual firms’ shares. However, since firms’ private information is not available to the market, all shares are traded at the same price per share, $P$.\footnote{In Section 4, firms are able to signal their type through the choice of capital structure; the share prices therefore depend on capital structure.} For an equity issuing firm, $P$ is its market capitalization. In order to raised one unit of funds, such a firm issues $1/P$ shares.

We also introduce returns on equity, $R$. If an intermediary buys $\alpha$ shares (in any firm) she invests $\alpha P$ in the first period and expects to get $\alpha P R$ in the second period. Unlike...
the straightforward relationship between price and returns to debt, \( p_r = 1 \), the return on equity is not a simple function of its price. The expected returns on equity are calculated by rational intermediaries who evaluate the average profitability of equity-financed firms; the set of firms which opt for equity financing is endogenous. The expression for the return on equity is derived below.

**Notation.** Let \( G(x) \) be expected returns conditional on returns being below \( x \):

\[
G(x) = E(\pi|\pi < x) = \frac{1}{F(x)} \int_x^\infty \pi f(\pi) d\pi,
\]

where \( f(\cdot) \) is the density function.

Let \( \pi^* \) denote the firm for which \( G(\pi^*) = \rho \) and suppose that equity is issued by all firms with profits below a certain level \( x \). Then \( G(x) \) is the expected profits of equity-financed firms. Therefore, \( \pi^* \) is the threshold level for which the average profit of equity-financed firms is still above intermediaries’ costs of funds, \( \rho \). In other words, \( \pi^* \) is the lowest \( r \) such that the average firm below \( r \) is worth investing in: \( E(\pi|\pi < r) = \rho \).

**Assumptions.** The following two assumptions simplify the structure of equilibria; under these assumptions there are at most two stable equilibria.

A1. Monotone hazard rate (MHR). \( (1 - F(x))/f(x) \) is a non-increasing function.
A2. \( x - G(x) \) is an increasing function.

In Section 3.3 we relax these assumptions and show that while the structure of equilibria remains similar, their number may increase.

### 2.2 Demand for finance

Consider the decision of a firm given the market prices of debt, \( p \), and equity, \( P \). The firm can finance the project either by borrowing one unit of funds or by issuing shares. If the firm borrows its payoff is \( \pi - r \). If the firm relies on equity it has to sell \( 1/P \) shares and its payoff is \( \pi - \pi/P \). Thus, the firm undertakes the project if \( \pi - \min\{r, \pi/P\} \geq 0 \). The capital structure, illustrated in Figure 1, is:

1. If \( P < 1 \), there is no equity financing. Good firms \( (\pi > r) \) borrow, other firms \( (\pi < r) \) do not undertake the project. Firms with \( \pi = r \) are indifferent between borrowing and not undertaking the project.

2. If \( P > 1 \), all firms undertake the project. Better firms \( (\pi > rP) \) borrow, other firms \( (\pi < rP) \) issue equity. The return on equity is \( R = \frac{G(rP)}{P} \). Firms with \( \pi = rP \) are indifferent between debt and equity financing.
Figure 1: The choice of capital structure. A firm with profit $\pi$ facing interest rate $r$ on debt and equity price $P$ chooses either debt or equity financing or no investment at all.

3. If $P = 1$, better firms ($\pi > r$) borrow, while other firms ($\pi \leq r$) are indifferent between issuing equity or not undertaking the project. The return on equity is $R = G(r)$.

There is no debt financing if $\pi < r$ and there is no equity financing if $P < 1$. The former condition is straightforward; the latter is related to the fact that each firm needs to raise a unit of funds. The firms cannot sell shares at prices below 1 because raising capital for the project requires giving out more than 100% of equity.

The market demand for debt finance (the total amount that companies want to raise through borrowing) is

$$ D(r, P) = 1 - F(r \max\{P, 1\}), $$

while the demand for equity finance is

$$ E(r, P) = \begin{cases} 
0, & P < 1 \\
[0, F(rP)], & P = 1 \\
F(rP), & P > 1 
\end{cases} $$

The total issue of shares is $E(r, P)/P$. 
The inverse demand functions $r(D, E)$ and $P(D, E)$ are:

$$
\begin{align*}
\text{If } E = 0 \text{ then } P &\in [0, 1] \text{ and } r \text{ solves } F(r) = 1 - D(r, P), \\
\text{If } E > 0 \text{ and } D + E < 1, \text{ then } P &\geq 1 \text{ and } r \text{ solves } F(r) = 1 - D(r, 1), \\
\text{If } E > 0 \text{ and } D + E = 1, \text{ then } P &\geq 1 \text{ and } r \text{ solves } F(r) = 1 - D(r, P), \\
\text{If } E > 0 \text{ and } D + E > 1, \text{ there are no finite prices.}
\end{align*}
$$

(1)

Note that for some values of $D$ and $E$ the price $P$ is not uniquely determined and can take on a continuum of values. It does not matter when $P < 1$ and equity is not issued. However, when $P > 1$ it may become a problem, since different values of $P$ result in different payoffs; higher price implies less outside equity issued and therefore higher payoffs of firms’ incumbents at the expense of outside investors. In all cases the dollar amount raised via equity issue is the same, but outsiders obtain either a large stake (if $P$ is close to 1) or a very small stake in the company (if $P$ is high). However this indeterminacy issue is not important; there are no equilibria with $P > 1$.

### 3 Analysis

#### 3.1 Perfect competition

As a benchmark, consider the case of perfectly competitive financial markets. When the intermediaries are price takers, the interest rate on debt is equal to the marginal cost of funds: $r = \rho$, and $p = 1\rho$. Equity finance is ruled out in equilibrium. If there were non-trivial equity issues, they should have also brought return $\rho$. Therefore $\rho = R = \frac{1}{P}G(rP)$. Using $r = \rho$, we obtain $rP = G(rP)$, contrary to the Assumption A2 that $G(x) < x$ for all $x$.

Thus, perfect competition implements the first best. All efficient firms ($\pi \geq \rho$) are financed, all inefficient firms are closed down. Equity is crowded out by debt because equity financing involves a dilution cost due to asymmetric information. This is exactly what a pecking order theory would imply in the absence of bankruptcy costs and costs of financial distress. This result is also similar to Akerlof’s analysis of the lemons’ problem. In equilibrium with competitive intermediaries, equity should bring the same returns as debt. But since only the best equity-financed firms $\pi = rP$ have returns equal to the interest rate on debt, the average equity-financed firm has quality below $rP$ and is not attractive to investors.
3.2 Structure of equilibria

In this section, we consider a market equilibrium where $N$ identical intermediaries interact strategically and solve for a Nash-Cournot equilibrium.\(^4\) Each intermediary chooses a two-dimensional strategy: how much to invest in debt $D_i$ and how much to invest in equity $E_i$. Essentially, the problem is similar to a multiproduct oligopoly: there are two products (debt and equity) and two prices ($p = 1/r$ and $P$). As in the conventional Cournot model, the intermediaries know the inverse demand functions $r(\sum_{i=1}^{N} D_i, \sum_{i=1}^{N} E_i)$ and $P(\sum_{i=1}^{N} D_i, \sum_{i=1}^{N} E_i)$ given by (1).

The payoﬀ of intermediary $i$ is

\[
(r - \rho)D_i + (R - \rho)E_i
\]

where $R = \frac{1}{p}G(rP)$. The intermediary chooses her investment strategy $(D_i, E_i)$ taking into account the strategies of other intermediaries: $D_{-i} = \sum_{j \neq i} D_j$, $E_{-i} = \sum_{j \neq i} E_j$.

To describe the structure of equilibria we introduce additional notation:

\[
N^D = \frac{1 - F(\pi^*)}{(\pi^* - \rho)f(\pi^*)}, \quad N^{ED} = 1 + N^D.
\]

**Proposition 1** Under the assumptions A1 and A2, the structure of equilibria is:

1. If $N \geq N^D$ there exists a (stable) equilibrium where only debt ﬁnancing is used ($P < 1$). The interest rate on debt $r = r^D(N)$ solves

\[
r - \rho = \frac{1 - F(r)}{Nf(r)}.
\]

Firms with $\pi \geq r$ borrow; ﬁrms with $\pi < r$ do not undertake the project.

2. If $N \leq N^{ED}$ there exists a (stable) equilibrium where both debt and equity are used. The price of equity is $P = 1$, the interest rate on debt solves

\[
r - G(r) = \frac{1 - F(r)}{(N - 1)f(r)}.
\]

Firms with $\pi \geq r$ borrow, ﬁrms with $\pi < r$ issue equity.

3. If $N \in (N^D, N^{ED})$ there exists an (unstable) equilibrium where both equity and debt are used. The price of equity is $P = 1$, the interest rate on debt is $r = \pi^*$. Firms with $\pi \geq r$ borrow, ﬁrms with $\pi < r$ use equity or do not undertake the project.

\(^4\) We extend a model of oligopolistic nonlinear pricing by Oren et al. (1983) to the multi-market case.
Figure 2: Equilibria structure. The graph shows the equilibrium interest rate on debt $r$ as a function of the number of financial intermediaries $N$. Whenever $r > \pi^*$, an average firm with $\pi < r$ is worth investing in ($G(r) = E(\pi|\pi < r) > \rho$); equity is issued in equilibrium. The dashed line denotes the unstable equilibria.

The comparative statics of equilibria with respect to market structure $N$ is illustrated in Figure 2. If the financial markets are perfectly competitive, $N \to \infty$, then equity is completely crowded out by debt and there exists a unique equilibrium which approximates the first best, $r^D(N) \to \rho$. If the financial markets are highly concentrated, then there exists a unique equilibrium in which all firms are financed: good firms borrow and bad firms issue equity.\(^5\) In the intermediate range of concentration, there are multiple equilibria.

The intuition behind the structure of equilibria is quite straightforward. First, because financial markets are imperfectly competitive, the interest rate on debt is set above investors’ cost of funds and the markup, $r - \rho$, decreases with $N$. Secondly, at every level of concentration, $N$, the interest rate on debt in equilibrium with both equity and debt is

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\(^5\)This result is consistent with Petersen and Rajan (1995) who show that a monopoly lender is more likely to form a relationship with the firm effectively obtaining a stake in firm’s future profits, similar to equity financing in our model.
higher than in equilibrium with debt only. The incentives to raise interest rates (through reduced lending) in an equilibrium without equity are lower; by increasing interest rates the intermediaries earn higher returns from borrowers but lose clients. In an equilibrium with both debt and equity higher interest rates generate higher returns on borrowers. The firms which stop borrowing do not drop out but switch to equity finance and bring additional profits to the intermediaries. Third, an equilibrium with equity exists if and only if the interest rate on debt is sufficiently high, \( r \geq \pi^* \), so that the average quality of equity-financed firms is also high, \( G(r) \geq G(\pi^*) = \rho \), while an equilibrium without equity requires the opposite, \( r \leq \pi^* \).

Multiple equilibria emerge as a result of the strategic complementarities generated by the return-on-equity externality. When an intermediary decides to lend more, the interest rate on debt is pushed down, as a result the best firms in the equity-financed pool switch to debt financing (to borrow from the very same intermediary). Once the best firms leave the pool its average quality declines. The incentives to invest in equity become lower and other intermediaries also prefer to switch to debt finance. This effect also explains why equilibria with \( r = \pi^* \) are unstable for \( N \in (N^D, N^{ED}) \). In such equilibria, every intermediary is indifferent between investing in equity and not investing. When an intermediary decides to lend more, others follow suit and the system moves to an equilibrium with debt and low interest rate, \( r^D(N) < \pi^* \). When an intermediary decides to cut lending, interest rates go up, and the average quality of equity-financed firms improves, others invest more in equity and less in debt. As a result, the market moves to the debt-equity equilibrium with a high interest rate, \( r^{ED}(N) > \pi^* \).

**Welfare analysis.** Whenever both equilibria coexist, the equilibrium without equity is more efficient in terms of social welfare. In the equilibrium without equity efficient firms with \( \pi \in (\rho, r^D(N)) \) are not financed resulting in the deadweight loss of \( \int_{\rho}^{r^D(N)} (\pi - \rho) f(\pi)d\pi \). In the equilibrium with equity, intermediaries cannot discriminate among firms within the equity pool, \( \pi \in [0, r^{ED}(N)] \). As a result, inefficient firms \( \pi \in [0, \rho) \) are also financed resulting in the deadweight loss of \( \int_{\rho}^{\rho}(\rho - \pi) f(\pi)d\pi \). The equilibrium without equity is less inefficient when the average firm that is denied financing should not have been financed in the first best: \( G(r^D(N)) < \rho \). This condition is equivalent to \( r^D(N) < \pi^* \) which follows from the existence of the equilibrium without equity.

Certainly, the welfare results should not be interpreted literally as a call to outlaw equity financing. First, we assume away all the costs of debt. Second, it is very likely that due to imperfections in the primary markets for funds for the intermediaries (banks), their costs of funds \( \rho \) is above social cost of funds. Therefore when equity finance helps
to implement projects with returns below $\rho$, it may actually be socially optimal.

**Example.** Consider $f(\pi) = 1/(\pi - \pi)$ for $\pi \in [\pi, \bar{\pi}]$. Uniform distribution satisfies both assumptions A1 and A2, so there can be at most two stable equilibria: one with debt, and the other one with debt and equity. Indeed, $G(\pi) = (\pi + \pi)/2, \pi^* = 2\rho - \pi, N^{ED} = (\pi - \rho)/(\rho - \pi), N^D = N^{ED} - 1$. The equilibrium with debt financing exists whenever $N \geq N^D$. In this equilibrium the interest rate is $r^D(N) = (N\pi + \pi)/(N+1)$; the deadweight loss is $(\pi - \rho)^2/ [2(N + 1)^2(\pi - \pi)]$. The equilibrium with debt and equity financing exists whenever $N \leq N^{ED}$, the interest rate is $r^{ED}(N) = ((N-1)\pi + 2\pi)/(N+1)$; the deadweight loss is $(\rho - \pi)^2/ [2(\pi - \pi)]$.

**Monopoly.** We do not consider the special case of a monopolistic intermediary. It is formally equivalent to the solution above at $N = 1$. The only difference is that there are no multiple equilibria: the monopolist chooses the one which is best for him. If $E\pi > \rho$, the monopolistic equilibrium is one with equity $P = 1, r = \pi$; actually there is no debt in this equilibrium. If $E\pi < \rho < \pi$, there is debt and no equity: $P < 1, r = r^D(1)$.

### 3.3 An example: Multiple equilibria with equity

This Section provides an example illustrating that once the assumptions A1 and A2 are relaxed, there can exist multiple equilibria of each type. In particular, it shows that there can be two equilibria with equity which differ in terms of both stock returns and amounts of equity financing.

Suppose that $\pi$ is distributed on $[\pi, \bar{\pi}]$ with the density function:

$$
 f(\pi) = \begin{cases} 
 0.6/(\pi - \pi), & \text{if } \pi \leq \pi < 0.75\bar{\pi} + 0.25\pi \\
 1.4/(\pi - \pi), & \text{if } 0.75\pi + 0.25\pi \leq \pi < 0.5\pi + 0.5\pi \\
 0.6/(\pi - \pi), & \text{if } 0.5\pi + 0.5\pi \leq \pi < 0.25\pi + 0.75\pi \\
 1.4/(\pi - \pi), & \text{if } 0.25\pi + 0.75\pi \leq \pi \leq \pi 
\end{cases}
$$

which does not satisfy the monotone hazard ratio property (see Figure 3).

The equilibrium with both debt and equity exists whenever a solution $r^{ED}(N)$ to (5) exists and satisfies $r^{ED}(N) > \pi^*$. Since the monotone hazard ratio property does not hold, the solution may not be unique. Figure 3 shows that for $N = 4$ there are two solutions $r_1^{ED}(N) = \pi + 0.40(\pi - \pi)$ and $r_2^{ED}(N) = \pi + 0.58(\pi - \pi)$. If $\rho < G(r_1^{ED}(N)) = \pi + 0.24(\pi - \pi)$ then either solution describes a stable equilibrium with debt and equity.\(^6\)

\(^6\)The equation (5) is essentially a first-order condition. One also needs to check whether $r_{1,2}^{ED}(N)$
The two stable equilibria have different interest rates on debt, \( r = r_{1}^{ED}(N) \) and \( r = r_{2}^{ED}(N) \) and, therefore, different average quality of equity-financed projects. The stock returns are also different \( R_1 = G(r_1^{ED}(N)) = \pi + 0.24(\pi - \bar{\pi}) \) and \( R_2 = G(r_2^{ED}(N)) = \pi + 0.32(\pi - \bar{\pi}) \). An equilibrium without equity also exists whenever \( \rho > \bar{\pi} + 0.21(\pi - \bar{\pi}) \).

## 4 Generalized model

The benchmark model above allows only a binary choice of capital structure: either debt or equity. In this section, we generalize the model in two directions. First, we allow for any combinations of debt and equity; second, we introduce agency costs.

When any combination of debt and equity is possible, the capital structure provides intermediaries with a more informative signal of the firm’s type. As better firms use more debt, intermediaries will pay a higher price for the stock of firms with lesser reliance on outside equity. In addition, if a firm’s value depends on the manager’s effort chosen after financing, then the firms that issue more outside equity will be priced further down by the stock market due to the agency costs.
Figure 4: Strategic complementarity in a setting with continuous choice of capital structure. In any equilibrium, better firms issue (weakly) more debt. As interest rate goes down, each firm reduces its reliance on equity finance. Therefore, for any given capital structure $\alpha$, investors expect a lower average return $\pi$ and therefore lower returns to equity investment.

Now the strategic interaction between the intermediaries becomes more complex, yet the strategic complementarity is still present. If one intermediary wants to lend more, the interest rates on debt are driven down, and each firm issues (weakly) less equity. Therefore given any capital structure $x$, the return on equity is now lower. As shown in Figure 4, the firm of a type $\pi$ that used to issue $x$ outside equity, now wants to issue $x - dx$ shares only; meanwhile $x$ shares are issued by a less productive firm $\pi - d\pi$. Hence, if one intermediary lends more, it provides other intermediaries with incentives to adjust their portfolios in favor of debt.

4.1 Setup

Technology. The firm’s profit $\pi$ is a random variable that takes “low” value $\pi^L > 0$ with probability $1-e$ and “high” value $\pi^H = \pi^L + \delta$ with probability $e$, where $\delta > 0$. Probability of the high outcome is identified with the firm’s effort, $e \in [0, 1]$. The cost of effort, $\delta C(e, \theta)$, depends on the firm’s type $\theta$ with $C_e \geq 0, C_e(0, \theta) = 0, C_{ee} > 0, C_{\theta} < 0$, and $C_{e\theta} < 0$. The firm’s productivity (type) $\theta$ is distributed on $[0, \infty]$ with c.d.f. $F(\theta)$. If the firm does not undertake the project, it receives the reservation payoff $u$.

We assume that an internal maximum exists: the parameters are such that the first best choice of effort (the one that solves $C_e(e, \theta) = 1$) is in $(0, 1)$ for all $\theta$. 

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The parameter $\delta$ is exogenous; it captures the importance of the agency problem: the higher $\delta$, the more severe moral hazard is.

Financial contracts. The firm needs to raise one unit of capital by issuing some combination of debt and equity. Given the price of debt $p = 1/r$ (dollars raised per dollar to be repaid) and the price of equity $P$ (in dollars per 100 percent of cash flow), the firm needs to borrow $D \geq 0$ and to issue $1 - \alpha$ shares, so that:

$$\begin{equation}
(1 - \alpha)P + D \geq 1. \tag{6}
\end{equation}$$

Once the profit $\pi_i, i = L, H$ is realized the firm pays $rD$ to creditors, $(1 - \alpha)(\pi_i - rD)$ to outside shareholders, and retains $\alpha(\pi_i - rD)$ for itself.

We assume that $r < \pi^L$ in equilibrium so there are no bankruptcies. Under limited liability we only need to consider contracts with $D \leq \pi^L$. Any contract with $D > \pi^L$ and $\alpha \in [0, 1]$ can be replicated by a contract with $D' = \pi^L$ and $\alpha' = \alpha \delta^{-1} [\pi^H - D]_+$.

Equilibrium. The definition of equilibrium extends the one in the basic model. An intermediary $i$ chooses a strategy $\{D_i, E_i(\cdot)\}$, where $D_i$ is the amount of money that $i$ invests in debt, and $E_i(\alpha)$ is the investment in equity of firms with capital structure $\alpha$. Each intermediary chooses her strategy given strategies of others and the inverse demand functions. The inverse demand functions determine the price of debt $1/r$ and the price of equity for each capital structure $P(\alpha)$ as functions of overall investment in debt and in equity of firms with each capital structure.

Hence, intermediary $i$ solves

$$\begin{equation}
\max_{D_i, E_i(\cdot)} \left( r - \rho \right) D_i + \int_0^1 E_i(\alpha) \left[ \frac{\pi^L + \delta v(\alpha, \theta(\alpha))}{P(\alpha)} - \rho \right] d\alpha
\end{equation}$$

where and $\theta(\alpha)$ is the equilibrium correspondence between firms' types and capital structures; $r$ and $P(\alpha)$ are inverse demand functions of $D_i + D_{-i}$ and $E_i(\alpha) + E_{-i}(\alpha)$.

4.2 Perfect competition

In the benchmark case of perfect competition, each intermediary sets the interest rate at $r = \rho$, there is no equity issued in equilibrium (the intuition is precisely as in the Section 3.1). Therefore each firm chooses effort to solve $\max_e [e - C(e, \theta)]$. For each type $\theta$ there is a respective effort level $e^*(\theta)$ and expected return $\pi = \pi^L + \delta e^*(\theta)$. The equilibrium is therefore equivalent to the one in the Section 3.1. It is socially optimal: both the project financing and effort are at their first best levels.
4.3 Equilibrium

**Effort choice.** The firm maximizes

\[ u = \alpha(\pi^L - rD)(1 - e) + \alpha(\pi^H - rD)e - \delta C(e, \theta) = \alpha \pi^L - \alpha rD + \delta v(\alpha, \theta) \quad (7) \]

where

\[ v(\alpha, \theta) = \max_{e \in [0, 1]} [\alpha e - C(e, \theta)] \]

It follows from the envelope theorem (or monotone comparative statics) that \( v_\alpha \geq 0, v_\theta \geq 0, v_{\alpha \alpha} \geq 0, v_{\alpha \theta} \geq 0 \) (the single crossing property holds). We denote by \( e^*(\alpha, \theta) \) the solution to

\[ \alpha = C(e, \theta). \]

The effort level \( e^*(\alpha, \theta) = v_\alpha(\alpha, \theta) \) (weakly) increases both in type \( \theta \) and in the share of equity \( \alpha \) kept by the firm.

Since incentives depend on the capital structure \( \alpha \), the investors will expect lower returns on equity of firms with higher outside equity, \( 1 - \alpha \). Therefore, the stock price \( P \) will depend on the capital structure \( \alpha \) as well. Since there are no bankruptcies, the rate of return on debt will be the same for all firms (a dollar invested in debt always brings \( r \) dollars whoever the borrower is).

Notice that (7) implies positive payoff for any firm with \( \alpha > 0 \) unlike the simple model of the previous section where all equity-issuing firms earn trivial rents.

**Demand for finance.** Given \( r \) and \( P(\alpha) \), a firm of type \( \theta \) chooses \( \alpha \in [0, 1] \) and \( D \geq 0 \) to maximize (7) subject to the constraint (6). The latter is always binding so we can solve for

\[ D = 1 - P(\alpha)(1 - \alpha). \]

Now the firm chooses \( \alpha \in [0, 1] \) to maximize

\[ u = \alpha(\pi^L - r + r(1 - \alpha)P(\alpha)) + \delta v(\alpha, \theta). \quad (8) \]

subject to

\[ [1 - 1/P(\alpha)]_+ \leq \alpha \leq 1, \]

and its participation constraint \( u \geq u_0 \).

For each \( \theta \) this problem has a solution \( \alpha^*(\theta, r, P(\cdot)) \). The single crossing condition \( v_{\alpha \theta} \geq 0 \) implies that more productive firms issue less outside equity: \( \partial \alpha^*/\partial \theta \geq 0 \). Let us denote by \( \theta_1 \) the solution to \( \alpha^*(\theta, r, P(\cdot)) = 1 \). All firms with \( \theta > \theta_1 \) are financed exclusively through debt.

It is also clear that the higher the interest rate \( r \), the less debt is issued: \( D^*_r \leq 0 \). This creates a strategic complementarity similar to one that drives the multiplicity of equilibrium in the basic model. Suppose that one intermediary decides to lend more.
This drives interest rate $r$ down. Therefore the intermediary’s expected return on stock with any given capital structure should decline. For $\alpha^*(\theta, r, P(\cdot))$ to remain constant an increase in $r$ must be accompanied with decrease in $\theta$. Investors know that under a higher interest $r$, a company that issues $1 - \alpha$ shares must be of a lower type (see Figure 4). The investors’ return on each share is $(1 - \alpha)(\pi^L - rD) + \delta(1 - \alpha) E \{ v(\alpha, \theta) | \alpha = \alpha^*(\theta, r, P(\cdot)) \}$ which increases in $\theta$. Thus, if one intermediary shifts from equity to debt, the incentives of others to invest in equity may also decline. Yet, unlike in the model of the previous section, we also need to check for change in stock price $P(\alpha)$ in response to lower interest rate. This is what is done below.

**Solving for inverse demand functions.** We will find $r$ and $P(\alpha)$—price of debt and price of stock of firm with capital structure $\alpha$—given total amount borrowed $D = \sum_i D_i$ and total funds raised via issues of equity by firms with the same capital structure $E(\alpha) = \sum_i E_i(\alpha) \geq 0$ (we naturally assume $E(1) = 0$). We will also solve for the (weakly monotonic) correspondence between firm’s type and capital structure $\alpha(\theta)$.

First, we can find the lowest type that is financed $\theta$. By definition, this type has capital structure $\alpha = \inf \{ \alpha : E(\alpha) > 0 \}$. The total amount of debt and equity financing must be equal to investment per firm (one unit) times the number of firms financed $(1 - F(\theta))$. Therefore

$$1 - F(\theta) = D + \int_\alpha^1 E(\alpha) d\alpha$$

(9)

The total number of firms with capital structure $\alpha$ issuing equity is $\frac{E(\alpha)d\alpha}{P(\alpha)(1 - \alpha)}$. Therefore we can find the correspondence between types and capital structures $\alpha(\theta)$ in equilibrium

$$F(\theta) = F(\theta) + \int_\alpha^{\alpha(\theta)} \frac{E(x)dx}{P(x)(1 - x)} \text{ for all } \theta \in (\theta, \theta_1)$$

(10)

The firm $\theta$ maximizes (8) with regard to $\alpha$. For all $\theta \in (\theta, \theta_1)$, the first order condition is as follows: $0 = \pi^L - r + \frac{d}{d\alpha} [\alpha(1 - \alpha)P(\alpha)] + \delta \nu_\alpha(\alpha, \theta)$. Integrating the latter with regard to $\theta$, we obtain the standard incentive compatibility constraint $u(\theta) = u(\theta_1) - \delta \int_\theta^{\theta_1} \nu_\theta(\alpha(\theta), \theta)d\theta$. Substituting (8) into both sides of the equation, we find the price of equity

$$P(\alpha(\theta)) = \frac{\pi^L - r - v(1, \theta_1) - \delta \int_\theta^{\theta_1} \nu_\theta(\alpha(\theta), \theta)d\theta}{\alpha(\theta)(1 - \alpha(\theta))}$$

(11)

for all $\theta \in (\theta, \theta_1)$.

There remain two conditions. First, the lowest participating type’s payoff is equal to her reservation utility:

$$u(\theta) = \pi^L - r + \delta v(1, \theta_1) - \delta \int_\theta^{\theta_1} \nu_\theta(\alpha(\theta), \theta)d\theta = u.$$  

(12)
Second, the lowest participating type’s capital structure is the lowest one financed:

\[ \alpha (\theta) = \alpha = \inf \{ \alpha : E(\alpha) > 0 \} \]  

Therefore, given the strategies of the investors \( D \) and \( E(\alpha) \), conditions (9)-(13) determine \( \alpha(\theta) \), \( P(\alpha) \), \( r \), \( \theta \), and \( \theta_1 \).

**Strategic complementarity.** For the brevity’s sake we do not discuss existence or uniqueness of equilibrium. We only establish that the general model has the same strategic complementarity property as the basic model. In particular as aggregate lending \( D \) increases, the return on equity investment \( R(\alpha) = E(\alpha) \frac{P^L+\delta v(\alpha,\theta)}{P(\alpha)} |\alpha = \alpha(\theta) \) declines for each capital structure \( \alpha \).

**Proposition 2** If \( E(\alpha) \) is constant, an increase in \( D \) results in lower \( r \) and lower return on equity investment \( R(\alpha) \) for each \( \alpha \in [0,1] \).

The proof is intuitive. First, it is easy to show that given equity investment \( E(\alpha) \) an increase in lending \( D \) cannot result in higher interest \( r \). Therefore, interest rate decreases, and each type borrows more issuing less outside equity. Thus, the curve \( 1 - \alpha(\theta) \) shifts (Figure 4) down. Since incentive compatibility implies that \( 1 - \alpha(\theta) \) is weakly decreasing, this curve also shifts left (i.e. for a given \( \alpha \) the type \( \theta \) that solves \( \alpha = \alpha(\theta) \) is now lower). Therefore \( \delta v(\alpha,\theta) \) decreases for a given \( \alpha \) and \( \theta \) such that \( \alpha = \alpha(\theta) \). Straightforward calculations show that as \( r \) decreases, the price of equity \( (11) \) must increase. Hence the return on equity decreases.

5 Extensions

**Alternative models of competition and market structures.** The results above are driven by the strategic complementarity in the choice of capital structure which is robust to many modelling assumptions. In particular, in Appendix B we move from Cournot to Bertrand model of competition. In that setting, intermediaries provide firms with differentiated financial services. The structure of equilibria remains very similar.

The main results are also robust once we assume away the symmetry between the financial intermediaries. In the model above, we assume that all the intermediaries have the same capacity to work in both debt and equity market. In other words, our stylized setup assumes the financial sector populated by equipotent universal banks that work in both debt and equity markets. Our results in fact extend to more general settings where there are both universal and specialized banks. If there are banks that only lend or buy
debt, banks that only buy equity, and banks that buy both, our results would still go through. As long as there are at least some banks that work in both markets and possess market power at least in the debt market, the strategic complementarity emerges which would generate the above structure of equilibria.

**Costs of financial distress.** For simplicity, we have assumed away costs of debt financing. If the project returns \( \pi \) are stochastic, and the firm faces either the incentive effects of debt overhang, or costs of financial distress and inefficient litigation, or costs of intermediated bank lending (as in Bolton and Freixas 2000), in any equilibrium some firms will use both debt and equity. Yet, as the structure of sorting does not change (better firms prefer more debt) and the main result will remain intact.

**Observed heterogeneity.** The pecking-order theory intuition that equity is suboptimal to debt predicts that better and safer firms use debt while firms that are denied debt financing or those that have already borrowed too much, resort to equity. In real world, there are other factors that determine the choice of capital structure. Given that equity finance involves a fixed cost of issue, larger firms are more likely to opt for stock market. At the same time, the larger firms may also have safer returns, so the sign of correlation between capital structure and source of funding may change. Still, our model would be relevant suggesting that among the firms with the same observed characteristics (such as size or sector), the ones that prefer debt are probably the ones with better prospects (this is consistent with empirical evidence surveyed in Myers 2001).

6 Conclusions

This paper studies a model of imperfect competition in financial markets with endogenous capital structure. The model builds on the pecking order theory of capital structure that assumes that firms are better informed about their growth opportunities than outside investors. An issue of equity sends investors a negative signal about the firm’s quality; the cost of equity financing is always higher than that of debt finance. Therefore, in the absence of the costs of financial distress the firms should finance their investment via internal funds or debt. Such a conclusion, however, hinges on the assumption that financial markets are perfectly competitive, so that all the imperfections of equity finance are automatically passed back to the firm in the form of a higher cost of capital.

We show that when financial markets are concentrated, this does not have to be the case: returns on equity and debt may differ. In the presence of oligopoly in financial
markets, some firms issue equity even if there are no costs associated with debt financing. The intuition is straightforward: oligopolistic financial intermediaries set interest rate on debt above their cost of funds. Hence, there are firms that would be financed in a perfectly-competitive economy but who cannot afford to borrow under oligopoly. The intermediaries are happy to finance these firms but do not like to lower interest rates for their more profitable debtors. Capital structure emerges as an effective tool for the (second degree) price discrimination: the most profitable firms prefer to be financed via debt rather than switch to equity.

An important implication of our analysis is the multiplicity of equilibria due to strategic interaction between oligopolistic financial intermediaries. The intermediaries’ portfolio choices are strategic complements: if one intermediary moves from debt- to equity-holding, others find it profitable to follow. When a large intermediary reduces lending and invests more in equity, the interest rate on debt goes up. Hence, some firms that used to be financed via debt have to switch to equity financing. As the marginal equity-financed firms are always better than the average equity-financed firms, this improves the expected returns on equity. Therefore investing in equity becomes relatively more attractive to other investors as well. The strategic complementarity also results in multiplicity of equilibria. It is important to note that there are no bubbles in the model: investors price each stock based on the rationally updated expectations of this stock’s returns. However, the returns are endogenous and are not uniquely determined given the multiplicity of equilibria. Our model suggest that there can be multiple equilibrium levels of stock prices based on fundamentals–and, therefore, sustainable in the long run. In order to fully explore the stock price dynamics a multi-period setup is needed, which is an exciting avenue for further research.

While we develop implications for the multiple equilibria in a public stock market, our model if taken literally is a model of an entrepreneur raising capital for a new project. While our intuition is not constrained to this case, a few formal extensions are due to make the argument more convincing: first, one would need to consider the case with assets in place prior to raising new funds, second, consider secondary markets for stocks, and third, introduce a conflict between management and initial shareholders (see Dybvig and Zender 1993 for the implications of the latter for the validity of the pecking order theory).
Appendix A: Proofs

Proof of Proposition 1.

Equilibria without equity.

Suppose that the equilibrium price of equity is low, \( P < 1 \). Then, no equity is issued and the equilibrium is a standard equilibrium of a single-product oligopoly. The inverse demand function implies \( dr/dD = 1/f(r) \). The first order condition for intermediary \( i \) is

\[
(r - \rho) - D_i/f(r) = 0
\]

Summing up across investors \( i = 1, \ldots, N \) and dividing by \( N \), we obtain (4). A1 implies that the right-hand side of (4) decreases in \( r \), hence there exists a unique solution denoted by \( r^d(N) \). Clearly, \( r^d(N) \) decreases with \( N \) and as \( N \to \infty \), the solution approaches the perfectly competitive one, \( r^d(N) \to \rho \).

This equilibrium exists whenever no intermediary could benefit from equity investment. Hence, \( E_i \) must be an optimal strategy for every \( i \). This is the case whenever the average quality of stock of firms that are not issuing debt is below the marginal cost of funds \( R = G(r) \leq \rho \). Thus, the equilibrium exists whenever \( r^d(N) \leq \pi^* \) which is equivalent to \( N \geq N_d \).

Equilibria with equity.

Suppose that \( P \geq 1 \), so firms with \( \pi \leq rP \) may issue equity. First, we will show that \( P > 1 \) cannot hold in equilibrium. Suppose that there is an equilibrium with \( P = \bar{P} > 1 \), and \( r = \bar{r} \). Then the following conditions should hold: (i) \( \sum_{i=1}^{N} D_i + \sum_{i=1}^{N} E_i = 1 \), (ii) \( \sum_{i=1}^{N} D_i = 1 - F(rP) \); (iii) there is an intermediary \( i \) who holds non-trivial equity position \( E_i > 0 \). We will argue that this intermediary will always have incentives to reduce her investment in equity. Even a small decrease in equity investment results in a discrete drop in stock price from \( \bar{P} \) to 1. Since the supply of debt funding \( \sum_{i=1}^{N} D_i \) does not change, the interest rate on debt will adjust accordingly to \( r = \bar{r} \bar{P} \) so that \( D_i + D_{-i} = 1 - F(\bar{r} \bar{P}) \) remains the same. Then the intermediary \( i \)'s payoff will increase: the first term in (2) will not change, while the second one will certainly increase: the decline in \( E_i \) is infinitesimal, while \( R \) jumps from \( G(\bar{r} \bar{P}) / \bar{P} \) to \( G(\bar{r} \bar{P}) \). Therefore equilibria with equity can only exist under \( P = 1 \).

There can be two types of equilibria \( P = 1 \): with full investment \( \sum_{i=1}^{N} D_i + \sum_{i=1}^{N} E_i = 1 \) and rationed investment \( \sum_{i=1}^{N} D_i + \sum_{i=1}^{N} E_i < 1 \). Let us first consider the equilibria with full investment. Then intermediary \( i \) maximizes \( rD_i + G(r)E_i \). The first-order condition is

\[
0 = r - G(r) - \frac{1}{f(r)} (D_i + G'(r)E_i)
\]
Summing up and using \( G'(x) = (x - G(x))f(x)/F(x) \) we obtain (5). Under the assumptions A1 and A2, the left-hand side increases in \( r \) while the right-hand side decreases in \( r \). Hence the solution \( r^{ED}(N) \) is unique and (as it is easy to show) decreases in \( N \).

Full investment \( D_i + E_i = 1 - (D_{-i} + E_{-i}) \) is an equilibrium strategy if and only if the maximum investment in equity is optimal. In other words, the return on equity should be above the cost of funds: \( R = G(r^{ED}(N)) \geq \rho \). In other words, \( r^{ED}(N) \geq \pi^* \) which is equivalent to \( N \leq N^{ED} \).

The last type of equilibria is the one with debt and equity where some firms do not undertake the project. This occurs when \( P = 1 \) but \( D_i + E_i < 1 - (D_{-i} + E_{-i}) \). The investors are indifferent about buying more equity. This may happen only if \( G(r) = \rho \), or \( r = \pi^* \). In this equilibrium, the first order condition for \( D_i \) is as follows:

\[
(r - \rho) - \frac{1}{f(r)} (D_i + G'(r)E_i) = 0
\]

Adding up for \( i = 1, ..., N \) we obtain

\[
N(r - \rho) - \frac{1 - F(r)}{f(r)} - (r - G(r)) \sum_{i=1}^{N} E_i / F(r) = 0
\]

After substituting \( r = \pi^* \)

\[
\sum_{i=1}^{N} E_i / F(\pi^*) = N - \frac{1}{r - \rho} \frac{1 - F(r)}{f(r)}
\]

The equilibrium exists whenever \( 0 < \sum_{i=1}^{N} E_i < F(\pi^*) \). One can easily check that the left inequality is equivalent to \( N > N^{D} \), while the right inequality is equivalent to condition is equivalent to \( N < N^{ED} \).

**Appendix B: Bertrand competition**

This Appendix introduces a model of Bertrand competition with differentiated goods. We extend, with modifications, the model of geographical banking competition by Degryse et al. (2009) to the case where intermediaries compete both in debt and equity.

Consider a unit circle at each point of which there is a unit mass of firms. The firms’ profitability, \( \pi_i \), is distributed with c.d.f. \( F(\cdot) \). There are \( N \) financial intermediaries uniformly located on the circle. The distance between a firm and an intermediary is a proxy for a disutility for a specific firm dealing with this particular intermediary; this disutility may arise due to a geographical or sectoral specialization of the intermediary or any other source of product/service differentiation.
Figure 5: Equilibria in Bertrand competition with differentiated goods. (a) is the equilibrium where only debt is issued. (b) and (c) are the equilibria where firms issue debt and equity. Areas $D_i$ and $E_i$ denote firms financed through intermediary $i$ via debt and equity, respectively. Other firms do not invest.

The intermediaries face a cost of funds $\rho$ and provide funding to the firms either through debt or through equity. Each intermediary $i$ sets the interest on the debt $r_i$ and the price of equity $P_i$. Each firm then decides whether to undertake a project or not, whether to finance it via debt or equity, and which intermediary to choose: each firm solves

$$\max_{i=1,\ldots,N} \{ \max \{ \pi - r_i, \pi - P_i \} - \tau x_i, 0 \}$$

where $\pi$ is the firm’s returns, $x_i$ is the distance to intermediary $i$, and $\tau$ is the transportation cost per unit of distance. The aggregate demand for debt and equity financing is therefore a function of prices $r_i$ and $P_i$ set by all the intermediaries.

We solve for a Nash equilibrium where each intermediary chooses $r_i$ and $P_i$ to maximize her profits given the strategies $r_{-i}$ and $P_{-i}$ of all the other intermediaries. We consider only symmetric equilibria where all the intermediaries use the same strategies $r_i = r_{-i}$ and $P_i = P_{-i}$. In these equilibria, all intermediaries compete only with their neighbors on the circle. Therefore, instead of studying the circle, it suffices to analyze price competition on an interval of length $1/N$. Let us denote the transportation costs between two neighboring intermediaries $\Delta = \tau/N$ and from now one measure the distances in terms of respective transportation costs.
The structure of equilibria depends on the intensity of competition (proxied by the number of intermediaries $N$). In the case of perfect competition ($1/N \to 0$), the interest rate is set at $r_i = \rho$ and there is no equity issue. If competition is strong ($1/N$ is small) then there are equilibria where firms issue debt but not equity – the intermediaries know that the equity issuing firms are on average only worth financing at the stock price below 1; but at these prices firms prefer not to issue equity at all. These equilibria are depicted in the Fig.5a.

If the competition is sufficiently weak ($1/N$ is large) then the equilibrium involves equity issue. Similarly to the Cournot model, the interest rate on debt is so high that the average firm that cannot afford issuing debt at these rates is sufficiently profitable. Hence, a stock price $P_i \geq 1$ pays off for the intermediary. There are two types of such equilibria: in the case shown in the Fig.5b, intermediaries compete only for the better firms (which are financed via debt); this equilibrium takes place if $r_i P_i = \tau / (2N) + r_i$. Fig.5c presents the other case where the intermediaries compete both for the debt-issuing firms and for the equity-issuing firms.

**Equilibria without equity.** First, we consider equilibria without equity (Fig.5a). Such equilibria exist whenever $r_i$ is sufficiently low, so that even for the firms with $x_i = 0$ there is no equity issue in equilibrium at share price $P_i = 1$. This condition implies that $\int_0^{r_i} (\pi - \rho) f(\pi) d\pi < 0$.

The intermediary 1’s optimization problem, given the interest rate of its neighbor $r_2$, is:

$$\max_{r_i} \int_0^{\Delta + r_2 - r_1} \frac{1}{2} (r_1 - \rho) (1 - F(r_1 + x)) dx$$

Differentiating this equation with respect to $r_1$ and imposing the symmetric equilibrium condition $r_1 = r_2 = r$, results in the following equation for the equilibrium interest rate:

$$0 = \int_0^{\Delta} (1 - F(r + x)) dx - \frac{1}{2} (r - \rho) \left(1 - F \left( r + \frac{\Delta}{2} \right) \right) - (r - \rho) F \left( \frac{\Delta}{2} \right) .$$

**Equilibrium with debt and equity** First consider the case where intermediaries compete in debt markets only (Fig.5b). This is the case whenever

$$r_1 (P_1 - 1) < \frac{1}{2} (\Delta + r_2 - r_1) \quad (14)$$

where $r_2$ is the interest rate set by the intermediary 2 and $r_1, P_1$ are the choices of the intermediary 1.
The intermediary 1’s total payoff is

\[ U = \int_{0}^{r_{1}P_{1}} \left( \frac{\pi}{P_{1}} - \rho \right) \frac{\pi (P_{1} - 1)}{P_{1}} f(\pi) d\pi + (r_{1} - \rho) \int_{r_{1}P_{1}}^{\Delta + r_{2} + r_{1}} \frac{\Delta + r_{2} - r_{1}}{2} f(\pi) d\pi + (r_{1} - \rho) \int_{r_{1}P_{1}}^{\Delta + r_{2} + r_{1}} \frac{\Delta + r_{2} - r_{1}}{2} f(\pi) d\pi. \]

The FOCs with respect to \( r \) and \( P \) are:

\[ 0 = U_{r} = \int_{r_{1}P_{1}}^{\Delta + r_{2} + r_{1}} f(\pi) d\pi (\pi - r_{1}) + \int_{r_{1}P_{1}}^{\Delta + r_{2} + r_{1}} \frac{\Delta + r_{2} - r_{1}}{2} - (r_{1} - \rho) \left[ \frac{1}{2} + \frac{1}{2} F \left( \frac{\Delta + r_{2} + r_{1}}{2} \right) - F(r_{1}P_{1}) \right] \]

\[ 0 = U_{P} = \frac{-1}{P^{2}} \int_{0}^{rP} \pi f(\pi) d\pi \left[ \pi + \rho - 2 \frac{\pi}{P} \right]. \]

As the equilibrium is symmetric \( r_{i} = r, \ P_{i} = P \), these equations become

\[ 0 = U_{r} = \int_{r_{P}}^{\pi + \Delta} \pi f(\pi) d\pi - r \left[ F \left( r + \frac{\Delta}{2} \right) - F(rP) \right] + \frac{\Delta}{2} \left[ 1 - F \left( r + \frac{\Delta}{2} \right) \right] - (r_{1} - \rho) \left[ \frac{1}{2} + \frac{1}{2} F \left( r + \frac{\Delta}{2} \right) - F(rP) \right] \]

\[ 0 = U_{P} = \int_{0}^{rP} \pi f(\pi) d\pi \left[ \pi + \rho - 2 \frac{\pi}{P} \right]. \]

The second equation implies as \( r \) increases, \( rP \) increases as well. The higher interest rate on debt, the more equity is issued in equilibrium.

Let us now consider the case where (14) does not hold, so the intermediaries compete in both debt and equity markets. Figure 6 represents a typical disequilibrium outcome.

\[ \Pi_{1} = \int_{0}^{\pi + \frac{\Delta}{1 - \rho + \frac{r_{1}}{1 + r_{1}}}} f(\pi) d\pi \left[ \frac{\pi}{P} - \rho \right] \pi \left( 1 - \frac{1}{P} \right) + \int_{\frac{\Delta}{1 - \rho + \frac{r_{1}}{1 + r_{1}}}}^{r_{2}P_{2}} f(\pi) d\pi \left[ \frac{\pi}{P} - \rho \right] \frac{1}{2} \left( \Delta + \pi \frac{P_{2} - \pi}{P} \right) + \int_{r_{1}P_{1}}^{r_{2}P_{2}} f(\pi) d\pi \left[ \frac{\pi}{P} - \rho \right] \frac{1}{2} \left( \Delta + r_{2} - \frac{\pi}{P} \right) + \int_{r_{1}P_{1}}^{\pi} f(\pi) d\pi \left[ r_{1} - \rho \right] \frac{1}{2} \left( \Delta + r_{2} - r \right) \]

Let us find the first order conditions:

\[ \frac{\partial U}{\partial r} = \frac{1}{2} \left[ 1 - F(rP) \right] (r - \rho) (\Delta + r_{2} - r). \]

Hence the \( r_{1} = \frac{1}{2} (\Delta + \rho + r_{2}) \). In the symmetric equilibrium \( r_{1,2} = \rho + \Delta \).
Let us now find the derivative with respect to the stock price:

\[
\frac{\partial U}{\partial (1/P_1)} = \int_0^{1-\frac{\Delta}{\pi+1-r_2}} \pi f(\pi) d\pi \left[ \pi + \rho - 2\frac{\pi}{P_1} \right] + \frac{1}{2} \int_{r_2 P_2}^{\Delta \pi P_2} \pi f(\pi) d\pi \left[ \Delta + \rho + \frac{\pi}{P_2} - 2\frac{\pi}{P_1} \right] +
\]

\[
+ \frac{1}{2} \int_{r_2 P_2}^{r_1 P_1} f(\pi) d\pi \left[ \Delta + \rho + r_2 - 2\frac{\pi}{P_1} \right].
\]

As we solve for the symmetric equilibrium, this becomes

\[
0 = \int_0^{\frac{\Delta P}{\pi P-\Delta}} \pi f(\pi) d\pi \left[ \pi + \rho - 2\frac{\pi}{P} \right] + \frac{1}{2} \int_{\frac{\Delta P}{\pi P-\Delta}}^{(\rho+\Delta) P} \pi f(\pi) d\pi \left[ \Delta + \rho - \frac{\pi}{P} \right].
\]

Thus, the main results are similar to those obtained in the Cournot model. The interest rate on debt increases with market concentration 1/N; the higher the interest rate, the more valuable the equity-issued firms are, which in turn results in broader equity markets (i.e. number of equity issuing firms increases in 1/N). The important difference from the Cournot setting is that the price of equity is above 1 (so that most equity issuers do earn positive rents).
Figure 7: Equilibria structure under Bertrand competition: uniform distribution. The bold line represents the equilibria with debt and equity, the thin line shows the equilibria with debt only. The circles denote the equilibrium with debt and equity with highest $N$ and the equilibrium with debt only with lowest $N$.

**Uniform distribution.** When the firm’s types are distributed uniformly on $[0, \pi]$ (assuming that $\pi$ is sufficiently high) the FOCs simplify to two quadratic equations and a cubic one. It is straightforward to verify that the equilibria with debt and equity exist whenever $r > \frac{3}{2} \rho$ while the equilibria with debt only exist whenever $r < 2 \rho$.

Figure 7 illustrates the equilibrium interest rate on debt as a function of number of intermediaries $N$ in the case when parameter values are: $\rho = 1$, $\tau = 6$, c.d.f. $F(\cdot)$ is uniform on $[0,6]$. The structure of equilibria is very similar to the Cournot model. First, equilibria with debt and equity exist as long as market is sufficiently concentrated (in this example, $N \leq 10$). Second, equilibria with debt only financing exist if market is sufficiently competition ($N \geq 5$). Third, there is a range of market structures (in this example, for $N \in [5, 10]$) when the two equilibria co-exist.
References


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